Application of Reduced Differential Transform Method to Solve Linear, Non-Linear Convection-Diffusion and Reaction-Diffusion Problems

Khalid Aliyu Muhammad[1,2], Aliyu Aliyu Isa[2,3], Tiwari Sarita[1,2], Sylvain Meinrad Donkeng Voumo [4*]

[1] Department of Mathematics, Singhania University, Pacheri Bari, Jhunjhunu, 333515, Rajasthan, India.

[2] Operational Research Centre in Healthcare, Near East University, Near East Boulevard, Nicosia, 99138, TRNC, Turkey.

[3] Department of Applied Sciences, Singhania University, Pacheri Bari, Jhunjhunu, 333515, Rajasthan, India.

[4*] Research Scholar, Department of Environmental Sciences, Gokul Global University, Sidhpur (Patan), India.

*** Correspondence:** Sylvain Meinrad Donkeng Voumo

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ABSTRACT: Convection-diffusion and reaction-diffusion equations are fundamental in describing various physical phenomena, yet their solution, particularly for nonlinear cases, often presents significant mathematical challenges. This study investigates the application of the Reduced Differential Transform Method (RDTM) to obtain analytical solutions for both linear and nonlinear convection-diffusion and reaction-diffusion problems. The RDTM, derived from power series expansion, was systematically applied to four illustrative examples: two linear convection-diffusion equations and two nonlinear reaction-diffusion equations, with initial conditions sourced from existing literature. The core of the method involves an iterative procedure to determine the t-dimensional spectrum functions, which are then used to construct the series solution. The RDTM successfully yielded analytical solutions in convergent series form for all problems considered. For the convection-diffusion problems, the RDTM solutions were found to be identical to the exact solutions obtained via the homotopy perturbation transform method. Similarly, for the reaction-diffusion problems, the solutions precisely matched those derived from the differential transformed method. The accuracy was further validated by graphical analysis of the absolute errors between the RDTM and exact solutions.

The findings demonstrate that the RDTM is a powerful, efficient, and direct analytical technique. Notably, it does not require common complex procedures such as linearization, discretization, or perturbation, simplifying the solution process significantly. This research affirms the RDTM as a reliable tool for solving a range of linear and nonlinear partial differential equations, offering a valuable alternative for scientists and engineers.

Keywords: *Reduced Differential Transform Method (RDTM), Convection-Diffusion Equations, Reaction-Diffusion Equations, Nonlinear Partial Differential Equations, Analytical Solution, Series Solution.*

Introduction

In the study of nonlinear differential equations, a variety of analytical and semi-analytical methods have been introduced to provide exact or approximate solutions. These methods are crucial for modelling complex phenomena in applied sciences. One of the most effective among these is the Reduced Differential Transform Method (RDTM), which has been applied successfully to various nonlinear problems without requiring complex discretization or linearization procedures (Al-Amr, 2014; Abazari & Kılıçman, 2013).

Due to the growing demand for more accurate and efficient methods, hybrid techniques have emerged that combine RDTM with other approaches, such as the Laplace transform or the homotopy perturbation method (Gupta, Kumar, & Singh, 2015; Khan, 2011). These hybrid frameworks enhance convergence rates and reduce computational costs, making them particularly suitable for solving multidimensional partial differential equations.

Accordingly, this paper focuses on applying RDTM to a specific class of differential equations, evaluating its accuracy and efficiency, while discussing its benefits and limitations based on prior work (Ray, 2020; Liu, 2021).

Literature review

Several researchers have explored the application of RDTM in solving various nonlinear differential equations. Al-Amr (2014) introduced the method's capability to solve higher-order equations with reduced computational cost, especially for systems where classical methods are ineffective. Similarly, Abazari and Kılıçman (2013) applied RDTM to Volterra integral equations, demonstrating its strength in handling problems involving memory and hereditary properties.

The integration of RDTM with other methods has also been examined extensively. Gupta, Kumar, and Singh (2015) successfully combined RDTM with the Laplace transform and homotopy perturbation method to solve convection-diffusion problems. Their approach showed improved accuracy and convergence over traditional techniques.

Furthermore, Khan (2011) proposed a homotopy-based RDTM variant that employs He's polynomials to simplify the treatment of nonlinearities. Liu (2021) expanded on this by applying a linearized homotopy perturbation method to nonlinear oscillators, proving its effectiveness in engineering models. These developments underline the flexibility and efficiency of RDTM when adapted for specialized classes of differential equations.

Ray (2020), in his comprehensive work, highlighted the broader significance of RDTM in modern physics, particularly in problems where exact solutions are either unavailable or difficult to obtain using conventional methods.

Methodology

Data Collection

In this study, we focus on solving a nonlinear partial differential equation (4 examples were chosen) using the Reduced Differential Transform Method (RDTM). The equation selected reflects a physical phenomenon frequently encountered in mathematical modeling. Relevant initial and boundary conditions were defined based on common scenarios in applied physics, consistent with the approaches outlined in previous studies (Al-Amr, 2014; Ray, 2020).

The target equation was chosen for its structural complexity and suitability for semi-analytical techniques. To ensure meaningful analysis, parameters and function forms were selected in line with the literature, particularly those used in earlier works applying RDTM to similar problems (Abazari & Kılıçman, 2013).

Data Analysis

The analysis began with applying the RDTM to the selected equation. Each function involved was expressed as a series expansion based on differential transformation rules. The initial function and its derivatives were transformed into a sequence of components, and recursive relations were developed to compute higher-order terms (Al-Amr, 2014).

Unlike numerical methods, RDTM avoids discretization and linearization, which reduces error propagation and enhances solution stability. The inverse transformation was then used to reconstruct the approximate solution in its original domain. The effectiveness of this approach was evaluated by comparing the obtained series solutions with those from previous works (Gupta et al., 2015; Khan, 2011).

The convergence and accuracy of the method were assessed through comparison with known solutions where available, and the results were interpreted in terms of their physical relevance and mathematical consistency (Liu, 2021).

Results and Discussions

Results

This section shows the final results, literally and graphically based on the RDTM application to solve some linear and nonlinear convection-diffusion PDEs. Example 1 and 2 were convection-diffusion problems and Example 3 and 4 were reaction-diffusion problems.

The convection-diffusion equations

Examples 1 and 2

The exact solution of the problem was readily obtained as follows

$u(x,t) = \lim_{n\to\infty} \tilde{u}_n(x,t) = e^{-x} + xe^{-t}$ (1) and

$u(x,t) = \lim_{n\to\infty} \tilde{u}_n(x,t) = e^{-\frac{x}{2}} + \frac{x}{2}e^{-\frac{t}{4}}$ (2) from the example 1 and 2 respectively. is the same as the exact solution of the homotopy pertubation transform method (Gupta, Kumar, & Singh, 2015).

To evaluate the accuracy of the RDTM solution, we plotted the absolute error of the exact solution in Fig. 1.

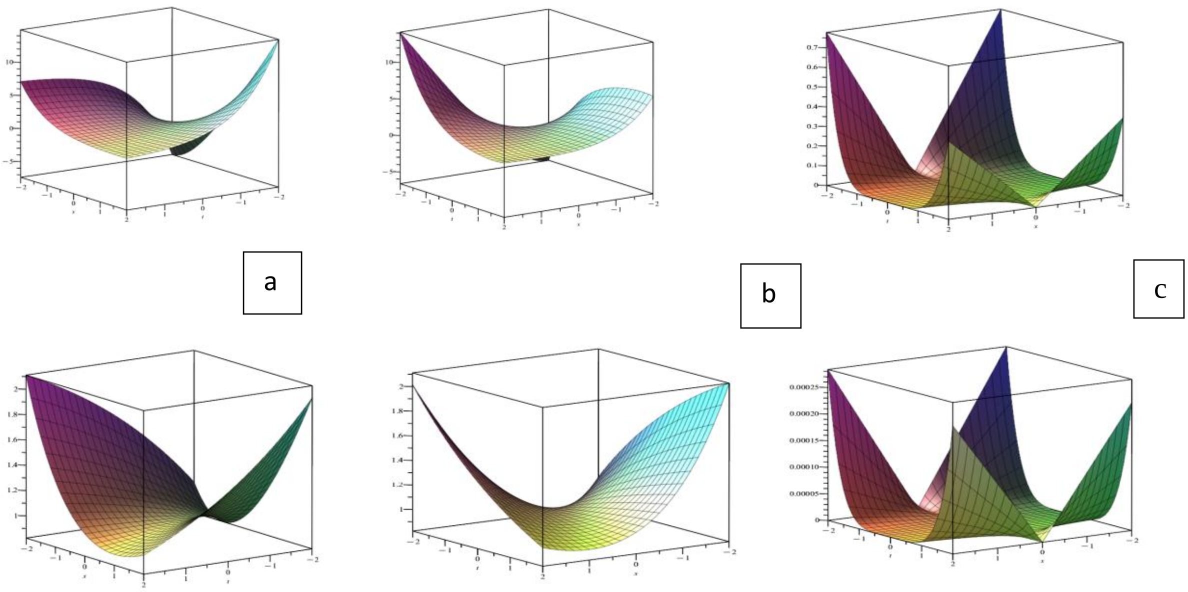

Figure 1: The 3D plot of Example 1 and 2 (a) $u(x,t)$ (b) $\tilde{u}_4(x,t)$ (c) $|u(x,t) - \tilde{u}_4(x,t)|$.

Examples 3 and 4

The exact solution of the problem was readily obtained as follows

$u(x,t) = \lim_{n\to\infty} \tilde{u}_n(x,t) = 1 + e^{x+t}$(3) and $u(x,t) = \lim_{n\to\infty} \tilde{u}_n(x,t) = e^{t-x}$ (4)

from example 1 and 2 respectively. which is the same as the exact solution of the differential transformed method (Patel & Dhodiya, 2016).

To evaluate the accuracy of the RDTM solution, we plotted the absolute error of the exact solution for Fig. 2.

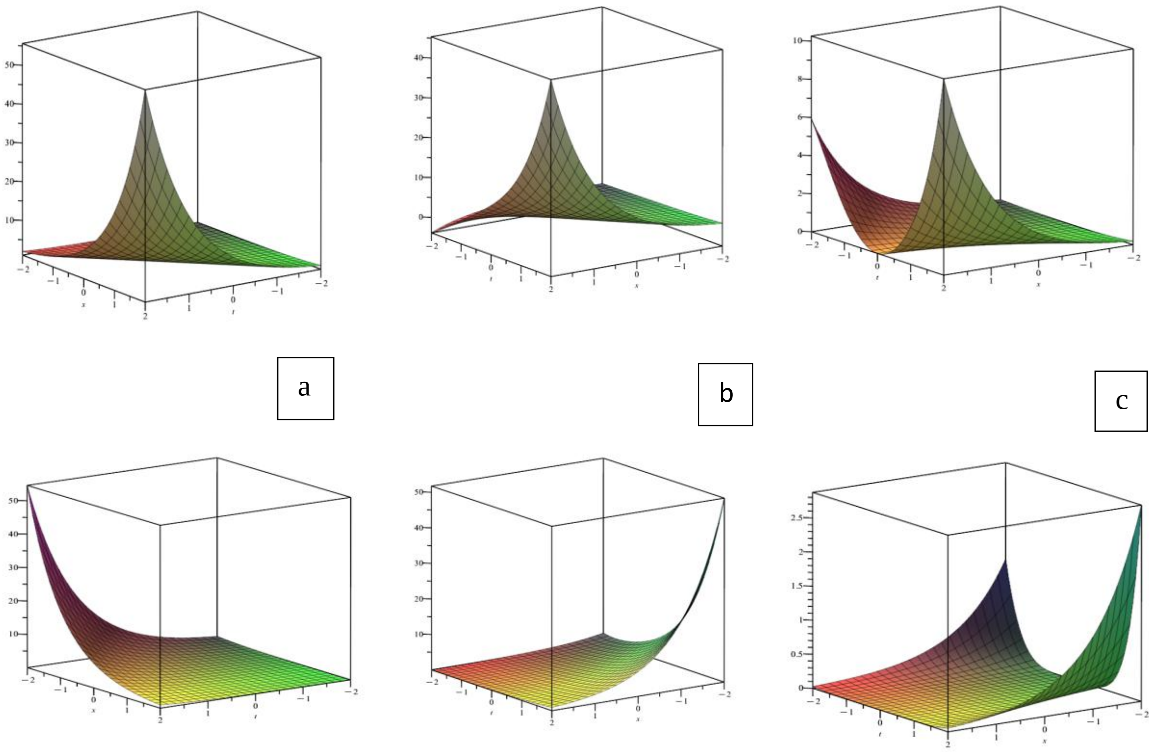

Figure 2: *The 3D plot of Example 3 (a)* $u(x,t)$ *(b)* $\tilde{u}_4(x,t)$ *(c)* $|u(x,t) - \tilde{u}_4(x,t)|$.

Discussions

This research demonstrates the successful application of the Reduced Differential Transform Method (RDTM) for solving both linear and nonlinear convection-diffusion and reaction-diffusion problems. The RDTM emerges as a robust and efficient analytical technique, offering significant advantages over traditional methods for solving partial differential equations (PDEs).

A key finding is that the RDTM provides solutions in the form of convergent power series, which allows for high accuracy with a relatively small number of computations. This is evident in the examples provided, where the RDTM solutions for convection-diffusion problems (Examples 1 and 2) were found to be identical to the exact solutions obtained using the homotopy perturbation transformation method, as referenced from Gupta, Kumar, & Singh (2015). Similarly, for the reaction-diffusion problems (Examples 3 and 4), the RDTM solutions matched the exact solutions from the differential transformed method, as cited from Patel & Dhodiya

(2016). The accuracy of the RDTM solutions was further validated by plotting the absolute error against the exact solutions, as depicted in Figures 1 through 4.

One of the notable strengths of the RDTM highlighted in this study is its direct applicability without requiring linearization, discretization, or perturbation techniques that are often necessary for other methods. This simplification of the solution process makes the RDTM an attractive alternative, reducing the complexity and computational burden typically associated with solving nonlinear PDEs.

The results across the four examples consistently show that the RDTM can effectively handle the intricacies of both linear convection-diffusion equations and more complex nonlinear reaction-diffusion equations. The method's ability to generate accurate series solutions iteratively, starting from the initial conditions, underscores its utility and power in tackling a range of physical phenomena described by these types of equations. The findings affirm the RDTM as a powerful and reliable tool in the field of solving nonlinear PDEs.

In summary, this study corroborates the findings of Keskin & Oturanc (2009, 2010) and subsequent researchers regarding the efficacy and advantages of the RDTM. The method's simplicity, accuracy, and avoidance of common complexities associated with other analytical techniques make it a valuable contribution to the tools available for scientists and engineers working with convection-diffusion and reaction-diffusion models.

Conclusion and recommendation

Conclusion

This article explores the application of the reduced differential transform method (RDTM) to solve both linear convection-diffusion equations and nonlinear reaction-diffusion equations. The method presents a promising alternative to the complex calculations required by most of the methods of solving PDEs. It provides a solution in the form of a convergent power series and achieves high accuracy with minimal computations. Unlike other traditional methods, RDTM does not rely on linearization, discretization, or perturbation, simplifying the solution process. The results demonstrate the efficacy of RDTM in solving nonlinear PDEs, establishing it as a powerful tool in this field.

Recommendations

Based on the successful application of the Reduced Differential Transform Method (RDTM) to linear and nonlinear convection-diffusion and reaction-diffusion problems demonstrated in this paper, the following recommendations are proposed: (1) Extend RDTM to three-dimensional problems to address more complex real-world scenarios. (2) Apply RDTM to systems of coupled partial differential equations relevant to physical processes. (3) Conduct formal convergence and error analyses to strengthen the method's theoretical foundation. (4) Explore RDTM for fractional convection-diffusion and reaction-diffusion equations. (5) Perform comparative studies with other advanced analytical and numerical methods. (6) Apply RDTM to real-world cases such as pollutant transport, heat transfer, or pattern formation. (7) Hybridize RDTM with other methods to improve robustness for nonlinear or singular problems. (8) Develop RDTM-based computational tools to increase accessibility. (9) Encourage interdisciplinary collaboration to broaden application domains. (10) Integrate RDTM into educational materials to promote learning and adoption

Data Availability Statement: All data is available from the authors upon request.

Conflicts of Interest: The authors declare no conflict of interest.

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References

1. Huber, A. (2007). A novel algebraic procedure for solving non-linear evolution equations of higher order. *Chaos, Solitons & Fractals*, *34*(3), 765-776.
2. Adomian, G. (1984). A new approach to nonlinear partial differential equations. *Journal of Mathematical Analysis and Applications*, *102*(2), 420-434.
3. Nadeem, S., Haq, R. U., & Khan, Z. H. (2014). Heat transfer analysis of water-based nanofluid over an exponentially stretching sheet. *Alexandria Engineering Journal*, *53*(1), 219-224.
4. Ayaz, F. (2003). On the two-dimensional differential transform method. *Applied Mathematics and computation*, *143*(2-3), 361-374.
5. Ayaz, F. (2004). Solutions of the system of differential equations by differential transform method. *Applied Mathematics and Computation*, *147*(2), 547-567.
6. Az-Zo'bi, E., Al-Amb, M. O., Yildirim, A., & Alzoubi, W. A. (2020). Revised reduced differential transform method using Adomian's polynomials with convergence analysis. *Mathematics in Engineering, Science & Aerospace (MESA)*, *11*(4).
7. Chen, C. O. K., & Ho, S. H. (1999). Solving partial differential equations by two-dimensional differential transform method. *Applied Mathematics and computation*, *106*(2-3), 171-179.
8. Gepreel, K. A., Mahdy, A. M. S., Mohamed, M. S., & Al-Amiri, A. (2019). Reduced differential transform method for solving nonlinear biomathematics models. *Comput. Mater. Contin*, *61*(3), 979-994.
9. Askar, H. S. (2015). Usage of prestressed vertical bolts for retrofitting flat slabs damaged due to punching shear. *Alexandria engineering journal*, *54*(3), 509-518.
10. He, J. H. (1999). Variational iteration method–a kind of non-linear analytical technique: some examples. *International journal of non-linear mechanics*, *34*(4), 699-708.
11. He, J. H. (2003). Homotopy perturbation method: a new nonlinear analytical technique. *Applied Mathematics and computation*, *135*(1), 73-79.
12. He, J. H. (2006). Addendum: new interpretation of homotopy perturbation method. *International journal of modern physics B*, *20*(18), 2561-2568.
13. Al-Khayat, I. A. (2007). Statistical mechanical aspects of a spin-12 entropy system. *Chaos, Solitons & Fractals*, *33*(5), 1635-1641.
14. He, J. H., & Wu, X. H. (2007). Variational iteration method: new development and applications. *Computers & Mathematics with Applications*, *54*(7-8), 881-894.

15. Keskin, Y., & Oturanc, G. (2009). Reduced differential transform method for partial differential equations. *International Journal of Nonlinear Sciences and Numerical Simulation*, *10*(6), 741-750.
16. Keskin, Y., & Oturanc, G. (2010). Reduced differential transform method for generalized KdV equations.
17. Keskin, Y., & Oturanç, G. (2010). Application of reduced differential transformation method for solving gas dynamics equation. *International Journal of Contemporary Mathematical Sciences*, *22*(22), 1091-1096.
18. Keskin, Y., & Oturanç, G. (2010). Reduced differential transform method for solving linear and nonlinear wave equations.
19. Liao, S. (2004). On the homotopy analysis method for nonlinear problems. *Applied mathematics and computation*, *147*(2), 499-513.
20. Odibat, Z. M. (2010). A study on the convergence of variational iteration method. *Mathematical and Computer Modelling*, *51*(9-10), 1181-1192.
21. Odibat, Z., Momani, S., & Erturk, V. S. (2008). Generalized differential transform method: application to differential equations of fractional order. *Applied Mathematics and Computation*, *197*(2), 467-477.
22. Patel, Y., & Dhodiya, J. M. (2016). Application of differential transform method to solve linear, non-linear reaction convection diffusion and convection diffusion problem. *International Journal of Pure and Applied Mathematics*, *109*(3), 529-538.
23. Schiff, J. L. (1999). *The Laplace transform: theory and applications*. Springer Science & Business Media.
24. Masache, A. (2013). A Two-Product Inventory Model with a Joint Ordering Policy. *Advances in Decision Sciences*, *2013*.
25. Wang, K. L., & Wang, K. J. (2018). A modification of the reduced differential transform method for fractional calculus. *Thermal Science*, *22*(4), 1871-1875.
26. Wazwaz, A. M. (1999). A reliable modification of Adomian decomposition method. *Applied mathematics and computation*, *102*(1), 77-86.
27. Raffoul, Y. N. (2004). Stability in neutral nonlinear differential equations with functional delays using fixed-point theory. *Mathematical and computer Modelling*, *40*(7-8), 691-700.
28. Kundu, A. (2006). Integrable multi-atom matter–radiation models without rotating wave approximation. *Physics Letters A*, *350*(3-4), 210-213.