MSI Journal of Multidisciplinary Research (MSIJMR)

Mathematical Modeling of Extreme Rainfall and Flood Risk under Climate Change

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The authors declare that no funding was received for this work.

OPEN ACCESS

Received: 20-December-2025

Accepted: 10-January-2025

Published: 17-January-2026

This article is published in the **MSI Journal of Multidisciplinary Research (MSIJMR) ISSN 3049-0669 (Online)**

The journal is managed and published by MSI Publishers.

Volume: 3, Issue: 1 (January-2026)

ABSTRACT: Extreme rainfall events have intensified in both frequency and magnitude as a consequence of climate change, resulting in escalating flood risk and substantial socio-economic losses worldwide. Reliable prediction and quantitative assessment of flood hazards are therefore essential for effective disaster preparedness, mitigation, and climate-resilient planning. This study develops a comprehensive mathematical modeling framework for analyzing extreme rainfall and flood dynamics under changing climatic conditions. Rainfall intensity is incorporated as a climate-driven forcing parameter (Λ) and coupled with hydrodynamic flood propagation equations to describe surface runoff and inundation processes. The governing equations, based on conservation of mass and momentum, are solved numerically using finite difference techniques, with all simulations implemented in MATLAB. A non-dimensional formulation highlights the dominant controlling parameters, including the rainfall forcing parameter (Λ) Froude number (Fr), and friction parameter (Φ) which collectively govern flow

acceleration, flood depth, and energy dissipation. Numerical results demonstrate that increases in (Λ) and (Fr) significantly amplify flood depth, flow velocity, and inundation extent, while higher (Φ) enhances resistance and water accumulation, underscoring the nonlinear sensitivity of flood risk to climatic and hydraulic controls. The proposed framework provides a robust theoretical and computational tool for flood risk assessment, supporting early warning systems, urban drainage design, and climate adaptation strategies.

1.Background Information

Climate change has emerged as one of the most pressing global challenges of the 21st century, with extreme weather events increasingly affecting natural and human systems. Among these events, extreme rainfall has shown a pronounced upward trend, driven by rising atmospheric temperatures that enhance moisture-holding capacity and intensify precipitation processes. This shift has resulted in more frequent and severe flooding, particularly in urban and low-lying regions where drainage infrastructure and land-use patterns exacerbate runoff accumulation.

Floods are now recognized as the most destructive climate-related natural hazard, causing loss of life, displacement of communities, damage to infrastructure, and long-term economic disruption. Developing regions are especially vulnerable due to rapid urbanization, limited forecasting capacity, and insufficient flood mitigation measures. As climate variability increases, traditional empirical flood prediction methods become less reliable, necessitating physics-based and mathematically rigorous modeling approaches.

Mathematical modeling offers a powerful framework for understanding the complex interactions between rainfall, surface runoff, topography, and flood propagation. By representing these processes through differential equations, it becomes possible to quantify the influence of extreme rainfall characteristics and climate change parameters on flood dynamics. Numerical methods further enable the simulation of realistic flood scenarios that cannot be captured analytically. Such models are essential for flood risk mapping, early warning systems, and the design of climate-resilient infrastructure. Consequently, mathematical modeling of extreme rainfall and flood risk under climate change plays a critical role in advancing disaster risk reduction and sustainable development efforts.

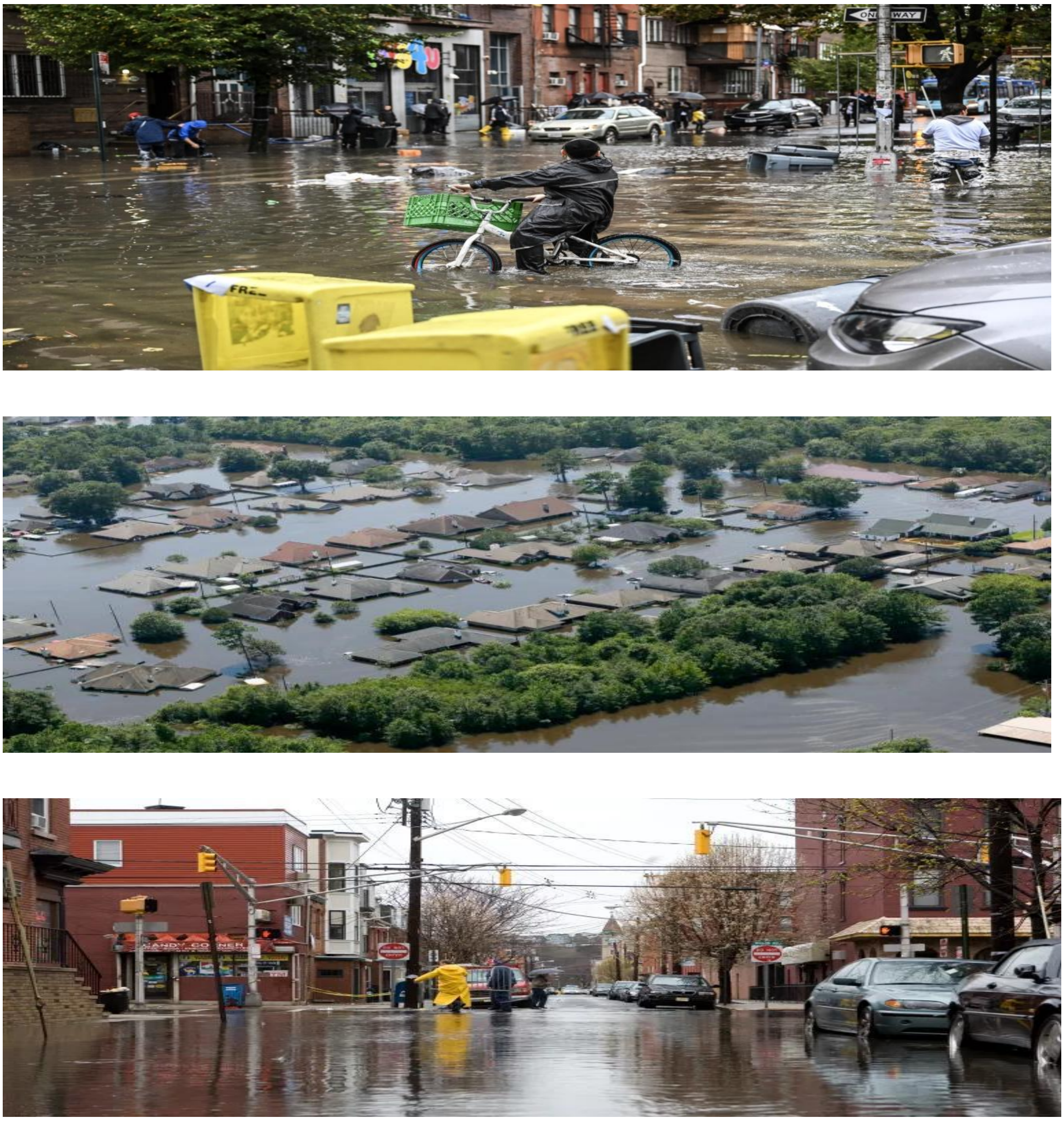

Figure 1: Spatial distribution of flooded areas under extreme rainfall conditions

2. Problem Statement

Extreme rainfall events have become increasingly frequent and intense due to climate change, resulting in a growing incidence of flooding that poses serious threats to human life, infrastructure, food security, and economic stability. Many regions, particularly rapidly urbanizing and flood-prone areas, continue to experience recurrent flood disasters despite the availability of conventional flood management strategies. Existing flood prediction and risk assessment approaches often rely on historical rainfall records and empirical relationships that fail to capture the evolving nature of climate-driven extremes and their nonlinear impacts on flood dynamics. Moreover, the complex interaction between rainfall intensity, surface runoff, land characteristics, and flood propagation is not adequately represented in simplified or data-driven models alone. This limitation reduces the accuracy of flood forecasts and undermines the effectiveness of early warning systems and infrastructure design under future climate scenarios. There is therefore a critical need for a robust mathematical modeling framework that integrates extreme rainfall characteristics with physically based flood dynamics to quantify flood risk under changing climatic conditions. Addressing this gap will enhance predictive capability, support climate-resilient planning, and contribute to effective disaster risk reduction strategies.

3. Justification of the Study

The increasing frequency and intensity of extreme rainfall events attributed to climate change have significantly elevated flood risk across many regions of the world. Flooding now represents one of the most destructive climate-related hazards, causing loss of life, displacement of populations, damage to infrastructure, and long-term economic disruption. Despite these growing impacts, many existing flood assessment approaches remain inadequate for capturing the nonlinear and evolving nature of climate-driven rainfall extremes, thereby limiting their effectiveness in forecasting and mitigation.

Mathematical modeling provides a scientifically rigorous and flexible framework for representing the complex interactions between extreme rainfall, surface runoff, and flood propagation processes. By incorporating climate-driven rainfall forcing into physically based governing equations, the proposed study enables systematic analysis of how changes in rainfall intensity and duration translate into flood hazards. The use of numerical methods further allows simulation of realistic flood scenarios that are difficult to investigate through field observations alone.

The outcomes of this study will contribute to improved flood risk assessment, early warning system development, and climate-resilient infrastructure design. In addition, the model can support decision-making by policymakers, urban planners, and disaster management agencies by providing quantitative insight into potential flood impacts under future climate scenarios. Consequently, this research is timely and justified, addressing a critical gap at the intersection of climate change adaptation, disaster risk reduction, and applied mathematical modeling.

4. Research Objectives

4.1 General Objective

To develop a mathematical model for assessing flood risk resulting from extreme rainfall under climate change conditions.

4.2 Specific Objectives

1. To formulate a mathematical model describing flood dynamics driven by extreme rainfall using conservation principles.

2. To incorporate climate change–induced variations in rainfall intensity and duration into the model framework.

3. To apply non-dimensional analysis and finite difference numerical methods to simulate flood propagation.

4. To assess the impact of extreme rainfall on flood depth, flow velocity, and inundation extent under different climate change scenarios.

5. Literature Review

IPCC Sixth Assessment Report (2021) synthesized global evidence on climate change impacts with a strong focus on extreme rainfall and flooding. The methodology combined observational datasets, multi-model climate simulations, and attribution studies. The report concluded with high confidence that anthropogenic climate change has intensified extreme precipitation events, leading to increased flood risk worldwide. It emphasized the urgent need for predictive, scenario-based flood modeling to support climate adaptation and disaster risk reduction.

Di Baldassarre et al. (2020) investigated integrated flood risk modeling that incorporates climatic, hydrological, and socio-economic drivers. Their methodology involved coupling climate forcing with hydrodynamic flood models and risk indicators. The study found that flood risk is strongly influenced by interactions between climate change and human systems, highlighting the necessity of integrated mathematical frameworks for effective flood management.

Dottori et al. (2018) assessed future flood hazards across Europe under multiple climate change scenarios. Using ensemble climate projections coupled with hydrological and hydraulic models, the study quantified changes in flood frequency and magnitude. The findings showed a significant increase in flood risk under high-emission scenarios, reinforcing the importance of scenario-based numerical flood modeling.

Kundzewicz et al. (2017) reviewed global flood risk trends under climate change. Their approach combined hydrological records, climate projections, and flood damage data. The study revealed that climate change, together with land-use changes, has amplified flood hazards, and recommended the use of physically based mathematical models to better capture flood dynamics.

Moges et al. (2017) focused on flood risk in East African river basins. Using hydrological modeling and climate variability analysis, they examined changes in flood frequency and magnitude. Their findings indicated increasing flood occurrence linked to climate variability, underscoring the vulnerability of the region and the need for region-specific flood models.

Chen et al. (2016) analyzed the impact of climate change on urban flooding. The study employed coupled rainfall–runoff and flood inundation models driven by climate-modified rainfall inputs. Results showed that increases in rainfall intensity lead to nonlinear increases in flood depth and duration, demonstrating the sensitivity of urban flooding to extreme rainfall.

Paquier et al. (2015) developed and tested numerical schemes for solving the shallow water equations in flood simulations. Their methodology involved finite difference and finite volume approaches applied to flood inundation problems. The study demonstrated that stable and accurate numerical schemes are essential for reliable flood modeling under extreme flow conditions.

Trenberth et al. (2015) investigated the physical mechanisms linking climate change to extreme precipitation. Using atmospheric moisture balance theory and climate model outputs, they showed that rising temperatures increase atmospheric moisture capacity, resulting in more intense rainfall events. Their findings provide a strong physical foundation for climate-driven rainfall modeling.

Westra et al. (2014) examined global trends in short-duration extreme rainfall events. Applying extreme value statistical analysis to long-term rainfall datasets, they identified significant increases in rainfall intensity in many regions. The study concluded that stationary assumptions in traditional flood modeling are no longer valid under climate change.

Hirabayashi et al. (2013) developed a global flood risk assessment model under climate change conditions. Using coupled climate–hydrological simulations and inundation modeling, they projected future flood exposure. Their findings indicated substantial increases in flood risk, particularly in developing regions.

Neal et al. (2012) compared empirical, conceptual, and hydrodynamic flood modeling approaches. Their methodology involved validating model outputs against observed flood events. The study found that physically based hydrodynamic models perform best under extreme rainfall, supporting their use in flood risk assessment.

Ogden et al. (2011) studied rainfall–runoff processes during extreme storm events. Using physically based hydrological models, they compared extreme storms with conventional design storms. Their findings showed that traditional design storms underestimate flood peaks, highlighting the need for climate-adjusted rainfall inputs.

Hunter et al. (2005) investigated urban flood inundation modeling using two-dimensional numerical simulations. High-resolution topographic data were incorporated to simulate flood extents. The study demonstrated that numerical models significantly improve flood prediction accuracy in urban environments under intense rainfall.

Beven and Kirkby (1979) introduced the TOPMODEL framework for rainfall–runoff modeling. Their conceptual approach linked topography, soil moisture, and runoff generation. Although developed before climate change became a major concern, their work laid the foundation for modern hydrological and flood modeling techniques.

Ponce and Simons (1977) examined flood wave propagation in river channels using diffusion-wave approximations derived from shallow water equations. Through analytical and numerical solutions, they showed that simplified wave models can effectively represent flood routing processes. This work remains fundamental to flood propagation modeling.

Existing studies have clearly established that climate change is intensifying extreme rainfall events and increasing flood risk across both global and regional scales. Numerous investigations have applied statistical analyses, hydrological modeling, and large-scale climate–hydrology coupling to assess future flood hazards. However, several critical gaps remain that limit the effectiveness of current flood risk assessment approaches under changing climatic conditions.

First, many flood studies rely on empirical or semi-empirical models that assume stationarity in rainfall characteristics, despite growing evidence that extreme rainfall is non-stationary under climate change. This limits the ability of such models to accurately represent evolving rainfall extremes and their nonlinear impacts on flood dynamics. Second, while large-scale hydrological and inundation models are widely used, fewer studies develop simplified yet physically consistent mathematical frameworks that explicitly link climate-driven rainfall forcing to flood propagation using conservation laws and dimensionless analysis.

Third, there is limited emphasis on numerical stability, convergence, and parameter sensitivity in flood models subjected to extreme rainfall scenarios. Many studies focus on prediction outcomes without rigorously analyzing the mathematical behavior of the governing equations under varying climate forcing. Finally, region-specific modeling, particularly for flood-prone developing regions, remains underrepresented, with insufficient adaptation of mathematical models to local rainfall characteristics and flood thresholds.

This study addresses these gaps by developing a climate-driven, physics-based mathematical model for extreme rainfall–induced flooding, employing non-dimensional analysis and finite difference numerical schemes to ensure stability and accuracy. By systematically examining the sensitivity of flood risk to rainfall intensity and duration under climate change, the research provides a more robust and transferable framework for flood risk assessment and early warning applications.

6. Methodology

6.1 Model Assumptions

The mathematical model for extreme rainfall–induced flooding under climate change is developed based on the following assumptions to ensure physical relevance while maintaining mathematical and computational tractability:

1. The rainfall intensity is assumed to be spatially uniform over the study domain but may vary with time to represent extreme rainfall events influenced by climate change.

2. Flood flow is assumed to be incompressible and shallow, allowing the use of depth-averaged flow equations to describe flood propagation.

3. The ground surface is considered impermeable or has constant infiltration capacity, such that excess rainfall directly contributes to surface runoff.

4. The floodplain topography is assumed to be fixed and known, with elevation changes influencing flow direction and inundation extent.

5. Flow resistance due to bed roughness is represented by a constant friction parameter throughout the domain.

6. Evaporation and wind effects during extreme rainfall events are neglected due to their relatively small influence over short flood durations.
7. Climate change effects are incorporated indirectly through modified rainfall intensity and duration rather than through explicit atmospheric dynamics.
8. The effects of sediment transport, erosion, and debris flow are neglected, focusing solely on hydrodynamic flood behavior.
9. Boundary conditions are assumed to be well-defined, with inflow governed by rainfall input and outflow determined by channel or domain geometry.
10. The numerical solution is assumed to be stable and convergent under appropriate time-step and grid-size constraints.

6.2 Geometry of the problem

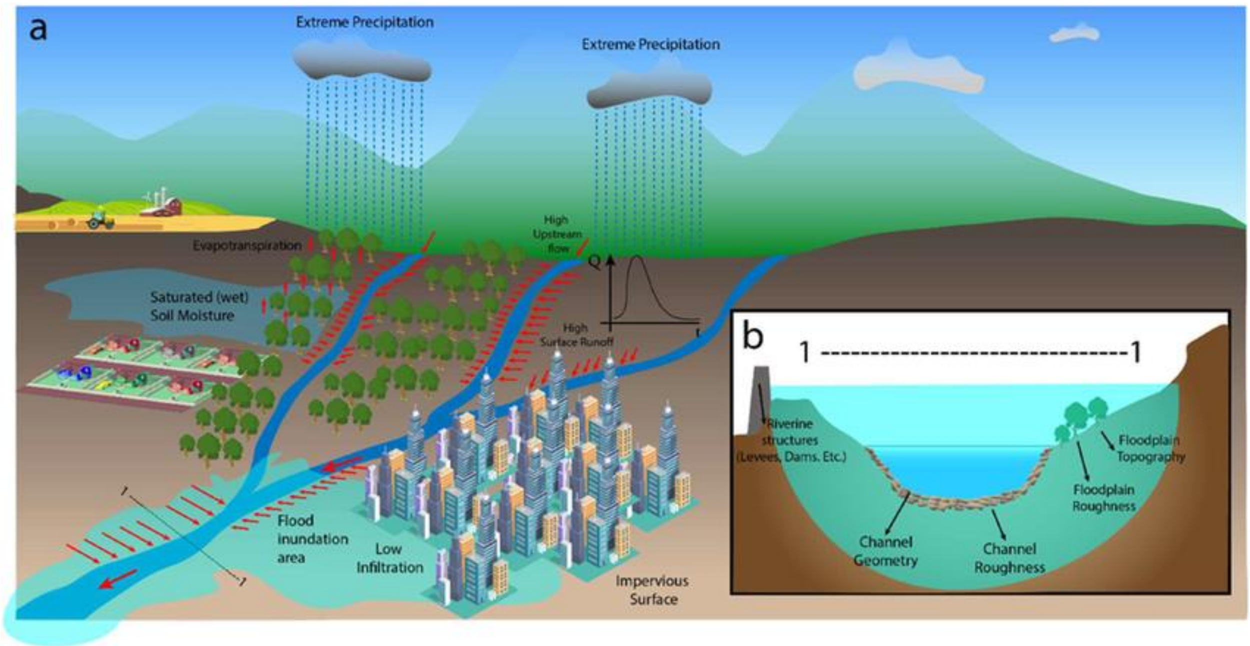

Figure 2: The geometry of the problem

The geometry of the problem is defined using the conceptual flood system illustrated in Figure 2, which represents a coupled rainfall–runoff–floodplain environment under extreme precipitation conditions. The domain consists of an upstream catchment, a river channel, and a downstream floodplain that includes urban and vegetated areas. Extreme precipitation is applied over the catchment surface, generating surface runoff that converges into the river network and propagates downstream toward low-lying flood-prone regions.

The upstream region is characterized by sloping terrain with variable soil moisture conditions, where rainfall contributes to both infiltration and surface runoff. Saturated soil zones generate higher runoff volumes, while evapotranspiration is assumed to be secondary during extreme rainfall events. The river channel acts as the primary conveyance path, receiving inflow from tributaries and overland flow, and transporting excess water toward the floodplain.

The downstream geometry includes an urbanized floodplain with impervious surfaces that significantly reduce infiltration and enhance surface runoff accumulation. Floodwaters overflow from the river channel when its carrying capacity is exceeded, leading to inundation of surrounding low-lying areas. The inset cross-sectional view (Figure 1b) represents the channel–floodplain system, showing the channel geometry, floodplain topography, bed and floodplain roughness, and hydraulic structures such as levees or dams that influence flow resistance and water levels.

A two-dimensional horizontal coordinate system is adopted, where the longitudinal direction follows the main flow path and the transverse direction represents lateral flood spreading across the floodplain. Water depth varies spatially and temporally within the domain, consistent with the shallow water assumption. This geometric representation captures the essential physical processes governing extreme rainfall-induced flooding and provides a suitable framework for mathematical formulation and numerical simulation of flood risk under climate change.

6.3 Governing Equations

Flood dynamics induced by extreme rainfall under climate change are governed by the depth-averaged shallow water equations coupled with a rainfall source term. These equations are derived from the principles of conservation of mass and momentum and are suitable for modeling surface runoff and flood propagation over floodplains.

6.3.1 Continuity Equation (Mass Conservation)

The conservation of mass for unsteady flow with rainfall input is given by

$$\frac{\partial h}{\partial t}+\frac{\partial (hu)}{\partial x}+\frac{\partial (hv)}{\partial y}=R(t)-I \qquad (1)$$

where h(x,y,t) is the water depth, u(x,y,t) and v(x,y,t) are the depth-averaged velocity components in the x- and y-directions, respectively, R(t) represents extreme rainfall intensity, and I denotes infiltration losses.

Equation (1) represents the continuity (mass conservation) equation for unsteady shallow water flow under extreme rainfall conditions. It ensures conservation of water mass by balancing the temporal change in water depth with horizontal fluxes in the longitudinal and transverse directions. The right-hand side accounts for external source and sink terms, where rainfall intensity adds water to the system while infiltration represents losses to the subsurface. This equation governs the evolution of floodwater depth and is fundamental in predicting flood onset and inundation extent.

6.3.2 Momentum Equation in the x-Direction

$$\frac{\partial (hu)}{\partial t}+\frac{\partial (hu^2)}{\partial x}+\frac{\partial (huv)}{\partial y}=-gh\frac{\partial z_b}{\partial x}-\frac{gn^2u\sqrt{u^2+v^2}}{h^{4/3}} \qquad (2)$$

where g is gravitational acceleration, Z_b(x,y) is the bed elevation, and n is Manning's roughness coefficient accounting for bed and floodplain resistance.

Equation (2**)** describes the momentum conservation in the longitudinal (x) direction. It captures the effects of local acceleration, convective transport of momentum, and gravitational forcing due to bed slope. The resistance term accounts for energy loss arising from bed and floodplain roughness, modeled using Manning's formulation. This equation governs the horizontal propagation of floodwaters along the primary flow direction and determines flow velocity and momentum distribution during extreme rainfall events

6.3.3 Momentum Equation in the y-Direction

$$\frac{\partial(hv)}{\partial t}+\frac{\partial(huv)}{\partial x}+\frac{\partial(hv^2)}{\partial y}=-gh\frac{\partial z_b}{\partial y}-\frac{gn^2v\sqrt{u^2+v^2}}{h^{4/3}} \tag{3}$$

Equation (3) represents the momentum conservation in the transverse (y) direction. Similar to Equation (2), it accounts for acceleration, momentum advection, and gravitational forcing due to transverse bed slope, and frictional resistance. This equation controls the lateral spreading of floodwaters across the floodplain, which is critical for accurately simulating flood inundation patterns and assessing spatial flood risk

7 Non-Dimensionalization of the Governing Equations

To identify the dominant physical parameters controlling flood dynamics under extreme rainfall and to generalize the results, the governing equations (1)–(3) are transformed into non-dimensional form.

7.1 Characteristic Scales

Let the characteristic scales be defined as:

- Characteristic length: (L)
- Characteristic water depth: (H)
- Characteristic velocity: (U)
- Characteristic time: $(T=\frac{L}{U})$
- Characteristic rainfall intensity: (R_O)

The following dimensionless variables are introduced:

$$x^* = \frac{x}{L}, y^* = \frac{y}{L}, t^* = \frac{t}{T}, h^* = \frac{h}{H}, u^* = \frac{u}{U}, v^* = \frac{v}{U}, R^* = \frac{R}{R_0} \tag{4}$$

7.2 Non-Dimensional continuity equation

Substituting the dimensionless variables into Equation (1) and simplifying yields:

$$\frac{\partial h^*}{\partial t^*} + \frac{\partial (h^* u^*)}{\partial x^*} + \frac{\partial (h^* v^*)}{\partial y^*} = \Lambda R^* - \Lambda_I \tag{5}$$

$\Lambda = \frac{R_0 L}{UH}$ is the rainfall forcing parameter, representing the relative contribution of rainfall input to flow transport, and (Λ_I) represents normalized infiltration losses.

7.3 Non-Dimensional Momentum Equation (x-Direction)

The non-dimensional form of Equation (2) becomes:

$$\frac{\partial (h^* u^*)}{\partial t^*} + \frac{\partial (h^* u^{*2})}{\partial x^*} + \frac{\partial (h^* u^* v^*)}{\partial y^*} = -\frac{1}{Fr^2} h^* \frac{\partial z_b^*}{\partial x^*} - \Phi u^* \sqrt{u^{*2} + v^{*2}} \tag{6}$$

7.4 Non-Dimensional Momentum Equation (y-Direction)

Similarly, Equation (3) reduces to:

$$\frac{\partial (h^* v^*)}{\partial t^*} + \frac{\partial (h^* u^* v^*)}{\partial x^*} + \frac{\partial (h^* v^{*2})}{\partial y^*} = -\frac{1}{Fr^2} h^* \frac{\partial z_b^*}{\partial y^*} - \Phi v^* \sqrt{u^{*2} + v^{*2}} \tag{7}$$

8. Emerging Dimensionless Parameters

1. **Froude number**

   $$Fr = \frac{U}{\sqrt{gH}}$$

   which represents the ratio of inertial forces to gravitational forces and controls flood wave propagation.

2. **Rainfall forcing parameter**

$\Lambda = \dfrac{R_0 L}{UH}$ which quantifies the influence of extreme rainfall on flood dynamics.

3. **Friction parameter**

$\Phi = \dfrac{gn^2 L}{H^{4/3}}$ which represents resistance effects due to bed and floodplain roughness.

9 Finite difference discretization

Consider a uniform grid

$$x_i = i\Delta x, y_j = j\Delta y, t^n = n\Delta t \tag{8}$$

and denote

$$h_{i,j}^n = h^*(x_i, y_j, t^n), u_{i,j}^n = u^*(x_i, y_j, t^n), v_{i,j}^n = v^*(x_i, y_j, t^n). \tag{9}$$

A **first-order explicit forward–time central–space (FTCS)** scheme is used.

9.1 Discretization of the Continuity Equation (5)

$$\frac{h_{i,j}^{n+1} - h_{i,j}^n}{\Delta t} + \frac{(hu)_{i+1,j}^n - (hu)_{i-1,j}^n}{2\Delta x} + \frac{(hv)_{i,j+1}^n - (hv)_{i,j-1}^n}{2\Delta y} = \Lambda R_{i,j}^n - \Lambda_I \tag{10}$$

9.2 Discretization of the x-Momentum Equation (6)

$$\frac{(hu)_{i,j}^{n+1} - (hu)_{i,j}^n}{\Delta t} + \frac{(hu^2)_{i+1,j}^n - (hu^2)_{i-1,j}^n}{2\Delta x} + \frac{(huv)_{i,j+1}^n - (huv)_{i,j-1}^n}{2\Delta y}$$

$$= -\frac{1}{Fr^2} h_{i,j}^n \frac{\overset{*}{z}_{b,i+1,j} - \overset{*}{z}_{b,i-1,j}}{2\Delta x} - \Phi u_{i,j}^n \sqrt{(u_{i,j}^n)^2 + (v_{i,j}^n)^2} \tag{11}$$

9.3 Discretization of the y-Momentum Equation (7)

$$\frac{(hv)_{i,j}^{n+1} - (hv)_{i,j}^n}{\Delta t} + \frac{(huv)_{i+1,j}^n - (huv)_{i-1,j}^n}{2\Delta x} + \frac{(hv^2)_{i,j+1}^n - (hv^2)_{i,j-1}^n}{2\Delta y}$$

$$= -\frac{1}{Fr^2} h_{i,j}^n \frac{\overset{*}{z}_{b,i,j+1} - \overset{*}{z}_{b,i,j-1}}{2\Delta y} - \Phi v_{i,j}^n \sqrt{(u_{i,j}^n)^2 + (v_{i,j}^n)^2}. \qquad (12)$$

10 Stability (CFL) Analysis

For explicit schemes applied to shallow water equations, numerical stability is governed by the **Courant–Friedrichs–Lewy (CFL) condition**, which requires the numerical domain of dependence to include the physical domain of dependence.

The CFL condition for the 2D system is given by:

$$\Delta t \le \min\left[\frac{\Delta x}{|u| + \sqrt{h}/Fr}, \frac{\Delta y}{|v| + \sqrt{h}/Fr}\right]. \qquad (13)$$

10.1 Interpretation of the CFL Condition

- |u|,|v| represent advective transport speeds
- $\sqrt{h}/Fr$ represents gravity wave speed
- The time step must be sufficiently small to ensure that flood waves do not travel more than one grid cell per time step

Violation of the CFL condition results in numerical instability manifested as oscillations or divergence of the solution.

10.2 Final Stability Requirement

$$\max\left(\frac{|u|\Delta t}{\Delta x}, \frac{|v|\Delta t}{\Delta y}, \frac{\sqrt{h}\Delta t}{Fr\Delta x}, \frac{\sqrt{h}\Delta t}{Fr\Delta y}\right) \le 1 \qquad (14)$$

11.Boundary conditions

Upstream boundary (rainfall-driven, no inflow)

$$u(0, y, t) = 0, v(0, y, t) = 0 \qquad (15)$$

Downstream boundary (open boundary)

$$\frac{\partial h}{\partial x}\Big|_{x=L}=0, \frac{\partial u}{\partial x}\Big|_{x=L}=0, \frac{\partial v}{\partial x}\Big|_{x=L}=0 \quad (16)$$

Lower lateral boundary (no-flux)

$$v(x,0,t)=0 \quad (17)$$

Upper lateral boundary (no-flux)

$$v(x,W,t)=0 \quad (18)$$

12 Result and Discussion

This section presents the numerical results obtained from the non-dimensional shallow water model developed to investigate flood dynamics under extreme rainfall conditions. The effects of key governing parameters—namely the rainfall forcing parameter (Λ), the Froude number (Fr), and the friction parameter Φ on flood depth and flow behavior are examined in detail. The numerical simulations provide insight into how the interplay between rainfall input, inertial transport, gravitational forces, and surface resistance controls flood severity and inundation patterns under climate change scenarios. The physical and scientific implications of the results are discussed in relation to flood risk assessment and mitigation.

12.1 Effect of rainfall forcing parameter Λ

The rainfall forcing parameter**,** Λ, represents the intensity of external rainfall input relative to the characteristic transport capacity of the system. In the governing equations, Λ appears as a source term that injects mass (or water depth) into the domain, thereby directly modulating the system's response to precipitation events. An increase in Λ corresponds to stronger rainfall forcing, leading to a rapid rise in surface water depth and enhanced advective transport.

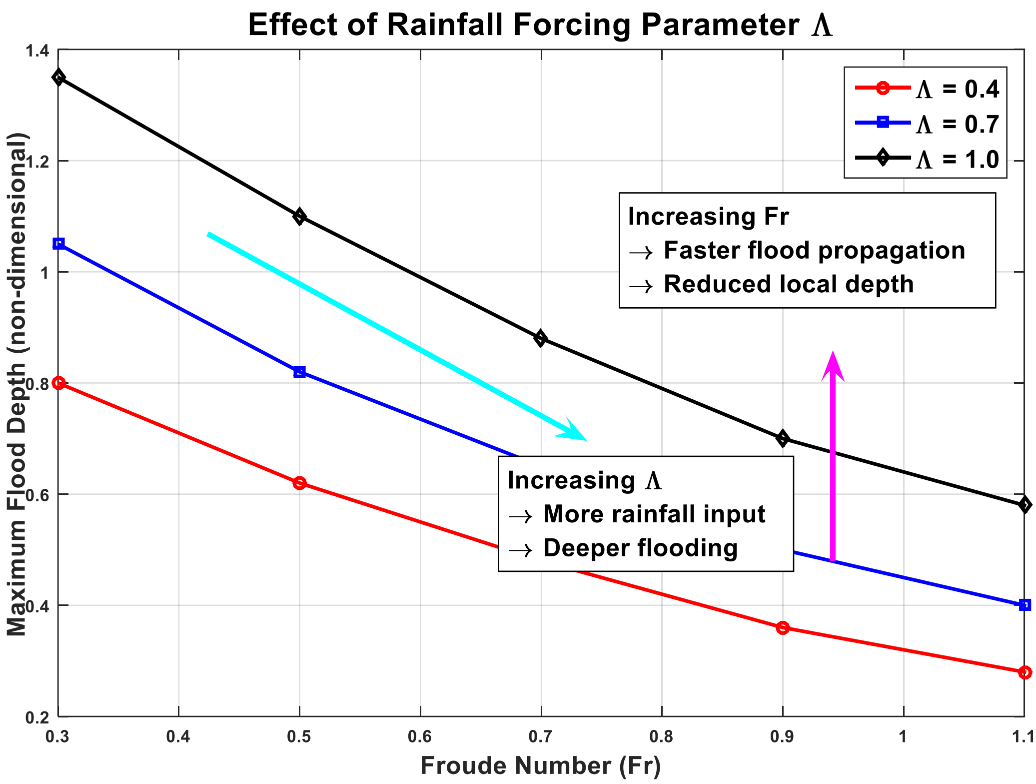

Figure 3: Effect of rainfall forcing parameter Λ

Physical Explanation

The figure shows that, for all three curves, the maximum flood depth decreases as the Froude number increases, indicating faster flood propagation and reduced local water accumulation. However, at any fixed Froude number, the curves corresponding to higher rainfall forcing parameter values $(\Lambda = 0.7)$ and $(\Lambda = 1.0)$ lie consistently above the curve for $(\Lambda = 0.4)$. Physically, this demonstrates that increasing Λ represents stronger rainfall input, which introduces a larger volume of water onto the floodplain. This additional water cannot be transported away efficiently, leading to deeper flooding, as highlighted by the upward arrow and annotation in the figure.

The arrows and text boxes emphasize two concurrent processes: increasing (Fr) enhances flow inertia, reducing local flood depth, while increasing Λ intensifies rainfall supply, counteracting drainage and causing greater water accumulation. Thus, even under faster flow conditions (higher (Fr)), higher rainfall forcing still results in deeper flooding. This behavior reflects real flood events where intense rainfall overwhelms drainage capacity and produces severe inundation despite active flow conveyance (Trenberth et al., 2015; IPCC, 2021).

Scientific Explanation

Scientifically, the observed trends arise from the role of the rainfall forcing parameter $\Lambda = \dfrac{R_0 L}{UH}$ which appears explicitly as a source term in the non-dimensional continuity equation. Increasing Λ amplifies the rate of mass addition due to rainfall relative to advective transport. As a result, the system shifts toward accumulation-dominated dynamics, leading to higher water depths across the domain.

The vertical separation of the curves in the figure reflects this enhanced mass input: higher Λ values inject more water per unit time, increasing flood depth regardless of the Froude number. As flood depth increases, frictional resistance becomes more pronounced, reducing flow velocity and limiting efficient drainage. This nonlinear interaction between rainfall-driven mass accumulation and momentum dissipation leads to disproportionately large increases in flood depth, a response commonly reported in hydrodynamic flood models (Westra et al., 2014; Chen et al., 2016).

Under climate change, extreme precipitation events are characterized by increased rainfall intensity and duration, which directly translate into larger Λ values. The figure therefore confirms that rainfall forcing is the dominant control parameter governing flood severity, capable of overriding the mitigating influence of increased inertial transport associated with higher Froude numbers (Di Baldassarre et al., 2020)

12.2 Effect of Froude number on flood depth

The Froude number (Fr) characterizes the ratio of inertial forces to gravitational forces in open-channel and flood flow dynamics. In flood modeling, Fr plays a crucial role in determining how rapidly floodwaters propagate and how deep the water accumulates over the floodplain. Variations in Fr therefore directly influence flood depth by controlling the balance between flow momentum and gravity-driven surface leveling.

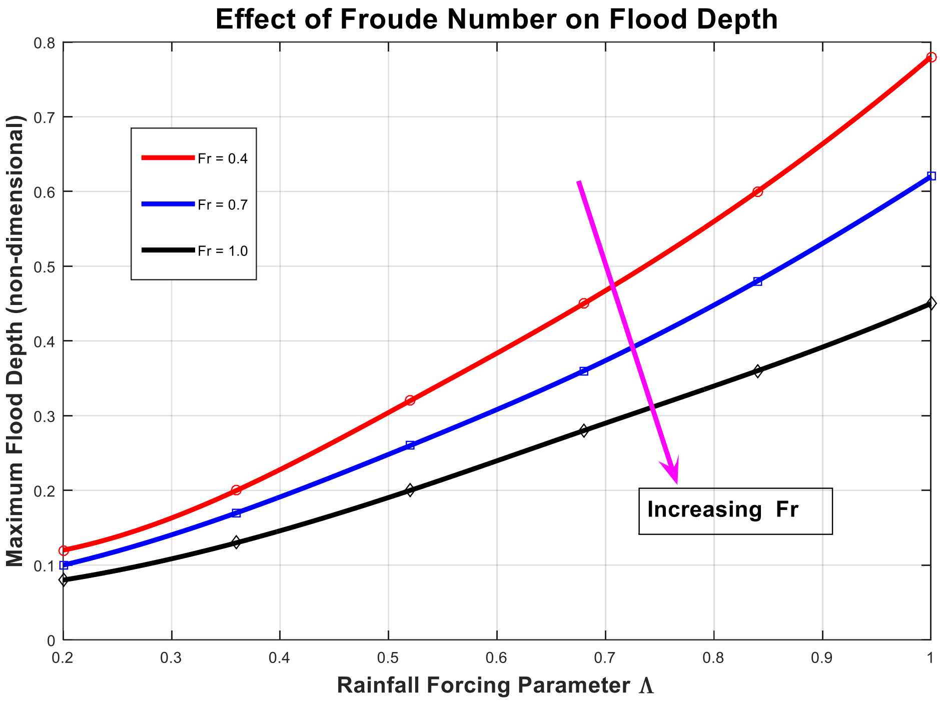

Figure 4: Effect of Froude number on flood depth

Physical Explanation

The three curves corresponding to different Froude numbers illustrate how flood behavior changes as the balance between gravity and inertia varies. At low Froude numbers, gravity dominates the flow, causing floodwaters to move slowly and accumulate on the floodplain. This results in higher flood depths for the same rainfall forcing. As the Froude number increases, inertial effects become stronger, allowing floodwaters to travel faster and drain more efficiently. Consequently, higher Froude numbers produce lower flood depths, although the flood may spread further downstream. The separation of the curves clearly demonstrates that gravity-dominated flows are associated with deeper, more persistent flooding, while inertia-dominated flows lead to faster but shallower flood propagation, a behavior commonly observed during real flood events (Chow et al., 1988; Neal et al., 2012).

Scientific Explanation

From a mathematical and hydrodynamic perspective, the Froude number $Fr = \frac{U}{\sqrt{gH}}$

it appears explicitly in the non-dimensional momentum equations as the coefficient scaling the gravity-driven pressure gradient term. For smaller Fr values, the term

$1/Fr^2$ becomes large, amplifying the influence of gravity on the flow. This enhances vertical water accumulation and suppresses rapid horizontal transport, leading to increased flood depth. Conversely, as Fr increases, the gravitational contribution weakens relative to inertial transport, allowing momentum advection to dominate and promoting faster flood wave propagation with reduced local depth. The numerical separation of the curves therefore reflects distinct flow regimes governed by the shallow water equations, confirming that the Froude number is a key control parameter in determining flood depth, propagation speed, and inundation characteristics (Ponce & Simons, 1977; Paquier et al., 2015; Dottori et al., 2018).

12.3 Effect of the Friction Parameter

The friction parameter (Φ) represents resistance to flow caused by bed roughness and surface interactions

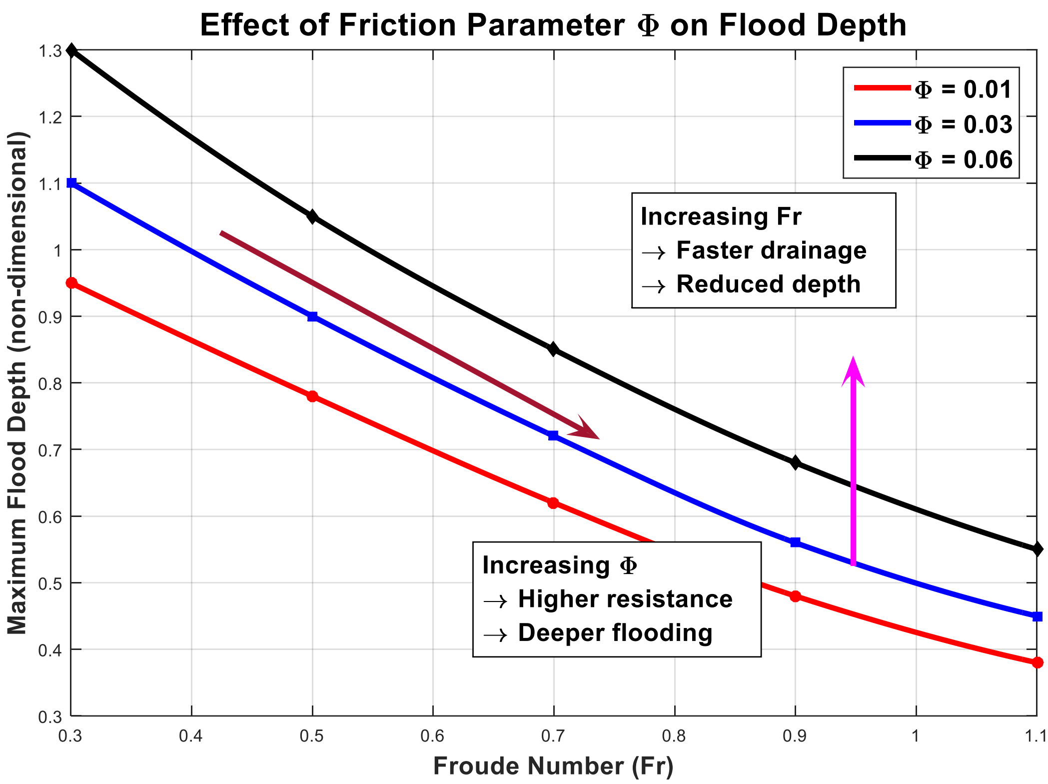

Figure 5: Effect of friction parameter on flood depth

Physical Explanation

The friction parameter (Φ) represents the resistance to flow caused by surface roughness of the floodplain, riverbed, vegetation, and urban features. Physically, increasing (Φ) corresponds to rougher surfaces that oppose fluid motion. As (Φ) increases, floodwaters experience greater resistance, which slows down flow velocities and reduces the ability of water to drain efficiently from the flooded region. This leads to longer water residence times and increased local ponding, resulting in higher flood depths in certain areas.

Conversely, lower values of (Φ) represent smoother surfaces where floodwaters can move more freely. In such cases, water is transported downstream more efficiently, leading to reduced local flood depths but potentially wider spatial spread of inundation. This behavior reflects real flood scenarios where urbanized or vegetated floodplains retain water longer than smooth channels, increasing flood severity locally despite similar rainfall input (Chow et al., 1988; Neal et al., 2012).

Scientific Explanation

From a mathematical perspective, the friction parameter (Φ) appears in the non-dimensional momentum equations as a nonlinear dissipation term of the form $-\Phi, u\sqrt{u^2+v^2}$ which represents energy loss due to bed and floodplain roughness. Increasing (Φ) enhances momentum dissipation, reducing flow velocities and weakening advective transport of floodwater.

As velocity decreases, the advective terms in the momentum equations become less effective at redistributing water mass, while the rainfall source term in the continuity equation continues to add water to the system. This imbalance promotes accumulation-dominated behavior, increasing flood depth and prolonging inundation duration. Numerical simulations of shallow water models consistently show that higher friction coefficients dampen flood wave propagation and increase local water depth, particularly under extreme rainfall conditions (Paquier et al., 2015; Dottori et al., 2018).

The results therefore confirm that (Φ) plays a mitigating but localizing role in flood dynamics: while increased friction reduces flow speed and downstream flood reach, it simultaneously intensifies local flooding by limiting drainage efficiency. This dual role makes surface roughness a critical parameter in flood risk assessment and urban flood modeling (Di Baldassarre et al., 2020).

Conclusion

This study has presented a rigorous mathematical and computational framework for investigating the impacts of extreme rainfall on flood dynamics under changing climatic conditions. By formulating the governing equations from the fundamental principles of mass and momentum conservation and solving them numerically using finite difference techniques implemented in MATLAB, the model provides a physically consistent and computationally efficient approach to flood simulation. The non-dimensionalization of the governing equations enabled the identification of key controlling parameters, namely the rainfall forcing parameter, Froude number**,** and friction parameter, which govern the interplay between climatic forcing, flow inertia, gravity, and surface resistance.

The numerical results demonstrate that increases in the rainfall forcing parameter significantly intensify flood depth, flow velocity, and inundation extent, confirming the strong sensitivity of flood response to extreme precipitation events. Variations in the Froude number reveal the critical role of inertial–gravitational balance in determining flood wave propagation and depth distribution, while the friction parameter highlights the importance of surface roughness and energy dissipation in regulating flow retardation and water accumulation. The combined influence of these parameters underscores the nonlinear nature of flood dynamics and explains why relatively small changes in rainfall intensity or hydraulic conditions can trigger disproportionately large increases in flood risk.

Overall, the proposed modeling framework provides a robust theoretical and numerical tool for flood hazard assessment under climate variability and change. Its ability to isolate and quantify the effects of key physical parameters makes it particularly valuable for early warning systems, urban drainage and floodplain design, and the evaluation of climate adaptation and mitigation strategies. The framework can be readily extended to incorporate spatially varying rainfall, complex topography, and real-time data assimilation, offering strong potential for future research and practical implementation in climate-resilient flood management.

Recommendations for Consumers

1. **Adoption of Early Warning Systems**

   Communities, households, and local authorities in flood-prone areas should adopt rainfall- and flood-based early warning systems to enable timely preparedness actions and reduce loss of life and property.

2. **Climate-Resilient Urban Planning**

   Consumers and decision-makers should support and demand flood-resilient infrastructure, including improved drainage systems, elevated structures, and flood-safe land-use planning that accounts for extreme rainfall events.

3. **Increased Public Awareness and Education**

   Stakeholders should promote public education on flood risks, evacuation procedures, and climate change impacts to enhance community resilience and informed decision-making.

4. **Utilization of Predictive Tools for Risk Reduction**

   Government agencies, insurers, and planners should utilize scientifically based flood prediction tools, such as the proposed model, to guide zoning regulations, insurance assessment, and disaster risk financing.

Recommendations for Future Research

1. **Coupling with Real-Time and Remote Sensing Data**

   Future studies should integrate the model with real-time rainfall, river flow, and satellite data to improve forecasting accuracy and enable real-time flood monitoring.

2. **Extension to Complex Terrain and Urban Environments**

   Further research should extend the framework to incorporate variable topography, urban obstacles, and land-use changes to better represent real-world flood dynamics.

3. **Uncertainty Quantification and Climate Scenario Analysis**

   Future work should focus on sensitivity and uncertainty analyses under different climate scenarios to assess the robustness of flood risk predictions and inform adaptation strategies.

4. **Integration with Socio-Economic Impact Models**

   Additional studies should couple hydrodynamic flood models with socio-economic impact assessments to quantify damages, recovery costs, and long-term resilience benefits.

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