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MATHEMATICAL MODELLING AND OPTIMIZATION OF ORE EXTRACTION PROCESSES IN MINING ENGINEERING

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ABSTRACT: In this study we show a mathematical model in mining systems for the processes of ore extraction in relation to mining industries in Papua New Guinea. Major mining operations such as Ok Tedi Mine, Porgera Gold Mine and Newmont Lihir Gold Mine have over decades produced sustainable quantities of gold and copper with contributing more than 70% of Papua New Guinea's export earnings in the recent years (Githiria, 2019). Regardless of this importance in economy, mining operations are affected frequently by variables such as equipment efficiency, reserve quantity, extraction rate and stochastic randomness that are a result of geological uncertainties are all integrated in the model. To optimize and predict the rate of ore extraction under geological and operational constraints we aim to develop a framework.

We formulate a system of differential equations to represent the depletion of ore over time (Øksendal, 2003) (Ross, 2014) and an additional stochastic component is then introduced to cater for random events in mining operations (Øksendal, 2003). Numerical simulations are conducted using Excel, to analyze system behavior under different extraction strategies, using an initial ore reserve of 1000 units with $k=0.3$, be the extraction coefficient. The deterministic model showed that from 1000 units the ore reserve went down exponentially to roughly 50 units after a span of 10 years. Whereas a fluctuating pattern was produced by the stochastic model with non-uniform ore quantities between 120 and 820 units which is due to the effect of uncertainty. By comparing aggressive, balanced and slow extraction strategies, the optimization simulation shows that the highest cumulative profit of K7710.00 was generated while extending the mine lifespan by more than 40% through the balanced extraction policy compared to the aggressive and slow extraction policy.

Findings indicate that optimal and well-designed extraction policies can improve recourse usage substantially while at the same time minimize operational cost (MacNeil & Dimitrakopoulos, 2017). The model illustrates that uncontrolled extraction leads to rapid depletion, on the contrary, sustainability is enhanced through controlled strategies.

This study enhances the role of applied mathematics in mining engineering by presenting a quantitative framework for making of decisions in ore extraction. The model can be calibrated using real data in mining to improve planning and operational efficiency in Papua New Guinea.

Keywords: *Sustainable mining; Stochastic Differential Models; Exponential Decay; Depletion; Randomness; ore extraction; Mining in Papua New Guinea.*

Introduction

In Papua New Guinea mining plays an important role in the economy, contributing to the nation's revenue and employment. While ensuring sustainability, efficient extraction of ore is key to maximize economic returns. However, ore extraction can be complicated and are influenced by geological contingency, operational constraints and equipment limitations.

This is where mathematical modelling provides a systematic approach to optimize and understand these processes (Goodfellow & Dimitrakopoulos, 2016).

Many mining operations like the landowners with mining leases up in Wau-Waria District, Morobe or large-scale mining industries rely on empirical or experienced based strategies rather than quantitative models, even though modern mining increasingly uses advanced technologies and numerical modelling for planning and optimization of operations in mine sites (Rademeyer, Minnitt, & Falcon, 2019), empirical methods remain as foundation for designing mining structures, particularly in geomechanics. Lack of usage in quantitative models can lead to inefficient extraction, growing costs and premature depletion of resources. Empirical experience provides the rule, like how steep a wall is, the quantitative models find the best solution among numerous possible combinations, something human experience alone cannot calculate.

There needs to be a Mathematical model that predicts extraction of ore over time, incorporates uncertainties and optimizes extraction strategies with the objective to develop a deterministic model for ore extraction and furthermore to extend the model to include stochastic variability and simulate extraction scenarios and finally analyze optimal extraction strategies

The scope of the study is that the model focuses on a single mining site with simplified extraction dynamics and theoretical or synthetic simulated data. The limitations include lack of real mining data and simplified data these may result in the model not capturing real world complications, but it provides a solid foundation for improvements in the future.

Literature Review

In mining engineering, mathematical modelling has been widely used to improve efficiency and planning (Githiria, 2019). Different approaches include stochastic processes, optimization techniques and differential equations (Ross, 2014) (Øksendal, 2003). Theses models hence will help describe how ore is being extracted and on how the system may behave over time.

There are many existing models that assumes ore extraction follows simple patterns, such as linear and exponential depletion. Probabilistic Models are used in some studies like the Markov Processes to account for uncertainties (Ross, 2014). Others test different strategies by relying on computer simulations (Goodfellow &

Dimitrakopoulos, 2016). However, for practical use, these models are often either too simple or too complicated.

Most of the existing models either tend to ignore uncertainty or which may be too difficult to be applied in real mining operations (MacNeil & Dimitrakopoulos, 2017). The aim of this study is to bridge the gap by developing a model that is realistic yet simple enough to use and understand, by combining deterministic and stochastic approaches.

Methodology

To simplify the model there are some assumptions to be made which are; The body of ore is finite, which means it will eventually deplete; The extraction rate depends how much ore is left. Equipment here we assume that it remains constant and lastly random interruptions or disturbances are also included to represent the uncertainty that occurs in mining industries. Classic differential equation approaches are used to model deterministic ore depletion (Ross, 2014) and using Wiener Processes the stochastic model uses stochastic differential equations. (Øksendal, 2003)

Parameters and variables

There are few key variables in the model:

- **Q(t)**: represents ore amount remaining at time *t*
- **t**: time (days or years)
- **E(t)**: extraction rate (this is how fast ore is being extracted)
- **k**: a constant that indicates how fast extraction happens
- **σ (sigma)**: represents uncertainty or random events
- **W_t**: a random process (used to model unpredictable changes)

Now firstly we have the deterministic model for which the idea present is that the more ore you have then you can extract it faster. This can be written as:

$$\frac{dQ}{dt} = -kQ$$

Here:

- $\frac{dQ}{dt}$ represents the speed of which ore decreases overtime
- Q is how much ore remaining
- k is how fast extraction takes place
- Lastly the negative sign indicates that ore is being removed

Equation solution

$$Q(t) = Q_0 \, e^{-kt}$$

Here is an exponential decay model where:

- Q(t): is the ore remaining in time t
- Q_0: this is the initial ore amount
- e: is a constant in mathematics (exponent)
- k: is the rate of extraction
- t: is the time

Note that in real mining operations initial ore amount is not guessed, it is estimated from data before or maybe at the start before the actual mining commences, mining engineers use core drilling samples, geological mapping, assay analysis and volume calculations of the ore body to estimate before mining starts.

Simulation setup

To simulate the model lets assume:

- initial ore amount = 1000 units
- rate of extraction = 0.3
- Time = 0 to 10 years

Using Excel to formulate the model we produce the data in the table below.

Time in years (t)	Ore Remaining Q(t)
0	1000
1	741
2	549
3	407
4	301
5	223
6	165
7	122
8	91
9	67
10	50

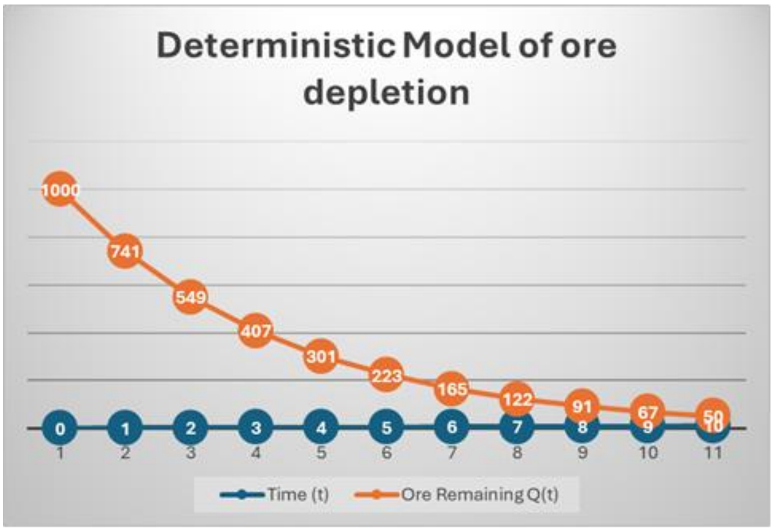

Figure 1: Using Excel, in the figures above you can see that ore depletes at an exponential decay rate with time in years for which the ore decreases steadily over time meaning there are no randomness.

Stochastic Model

In real minig operations, things will not be perfectly smooth so we have to add random variables. Hence:

$$dQ(t) = -kQ(t)dt + \sigma Q(t)\, dW_t$$

where:

- $-kQ(t)dt$: is normal extraction like before
- $\sigma Q(t)\, dW_t$: is random change (the uncertainty)

- σ(sigma): this is how strong randomness is
- dW_t : random fluctuation which may be sudden changes in geology

This means that extraction therefore is mostly predictable but then again sometimes it fluctuates randomly meaning it goes up and down at random.

Simulation Setup

Just like in the above simulation now we introduce randomness to be:

- σ(sigma) = 0.2

Time in years(t)	Ore Remaining Q(t)
0	1000
1	780
2	820
3	610
4	670
5	520
6	480
7	350
8	300

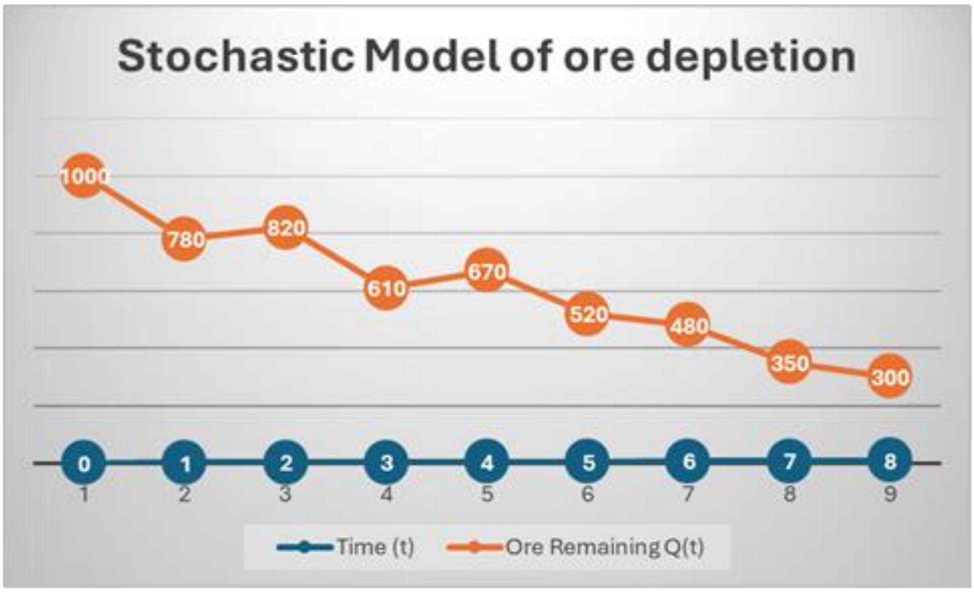

Figure 2: Using Excel to simulate, the figure now shows how ore reserve fluctuates over time due to random events that affect ore extraction.

Optimization Model

In the optimization models the aim is to make the most profit hence:

$$\max \int_0^T [P \cdot E(t) - C(E(t))] dt$$

where here we have:

- P = the price of ore
- E(t) = the rate of extraction
- C(E) = The Cost of extraction
- T = time in total

This equation simply means that profit is equal the difference between the money earned and the cost subject to: $\frac{dy}{dx} = -E(t)$, this is to ensure that the extraction, overtime, reduces the ore.

Simulation Setup

To simulate the idea here is to maximize profit over time, now profit depends on how much ore is being extracted and how much it costs to actually extract it. So, we assume:

- Initial ore amount = 1000 units
- Time = 10 years
- Price/unit of ore = 10 kina
- Cost function:
  $C(E) = 2E^2$ whereas the cost increases if you extract too fast.

Now we compare 3 different strategies, aggressive extraction where we extract too quickly with high and short-term gains, moderate or balanced extraction where we have controlled extraction that is sustainable and finally slow extraction where we are very careful with long mine life but low income.

Aggressive

Year	Ore Left	Extraction	Profit(K)
0	1000	400	2000
1	600	300	1200
2	300	200	600
3	100	100	200
4	0	0	0

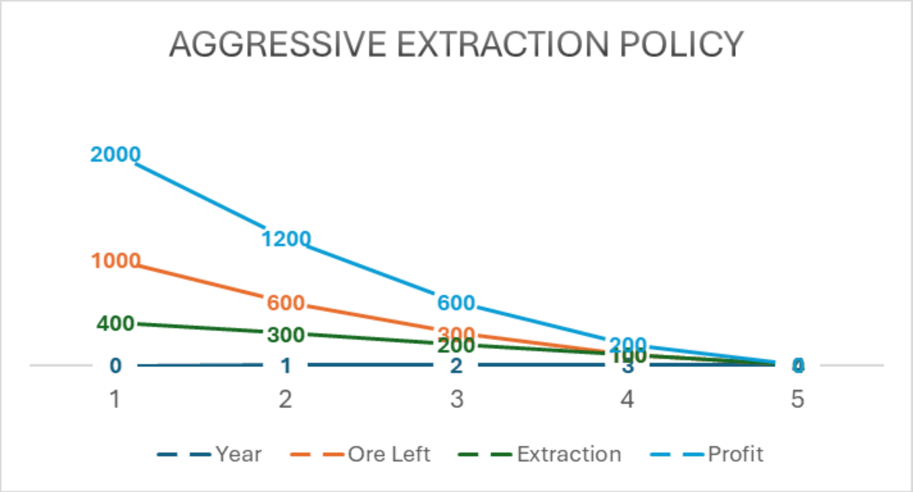

Figure 3: The overall life span of the mine here is only up to 4 years and generated a cumulative profit of K4000.00

Moderate

Year	Ore Left	Extraction	Profit(K)
0	1000	200	1600
1	800	180	1450
2	620	160	1300
3	460	140	1100
4	320	120	900
5	200	100	700
6	100	80	500
7	20	20	160

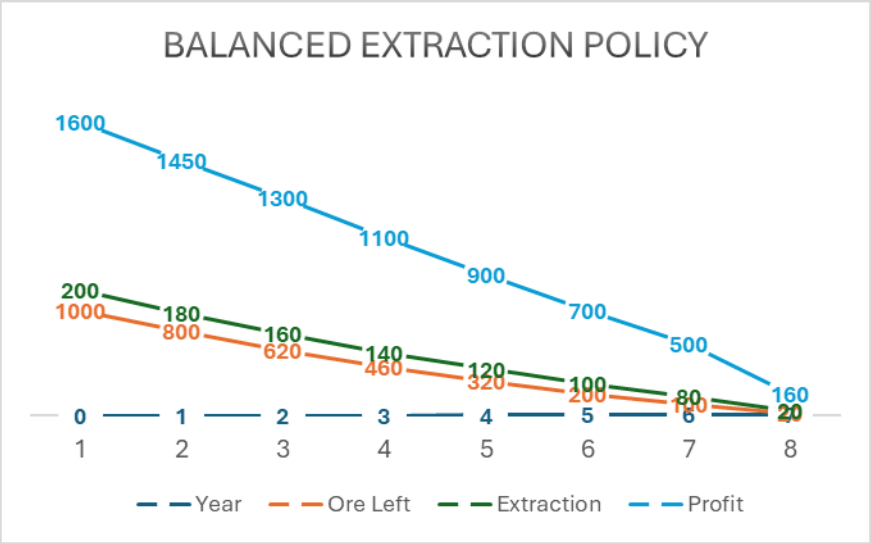

Figure 4: the overall lifespan of the mine site is 7 years and generated a cummilative profit of K7710.00. we can observe the the profit is stable over time

Slow

Year	Ore Left	Extraction	Profit(K)
0	1000	100	800
1	900	90	720
2	810	80	640
3	730	70	560
4	660	60	480
5	600	50	400
6	550	50	400

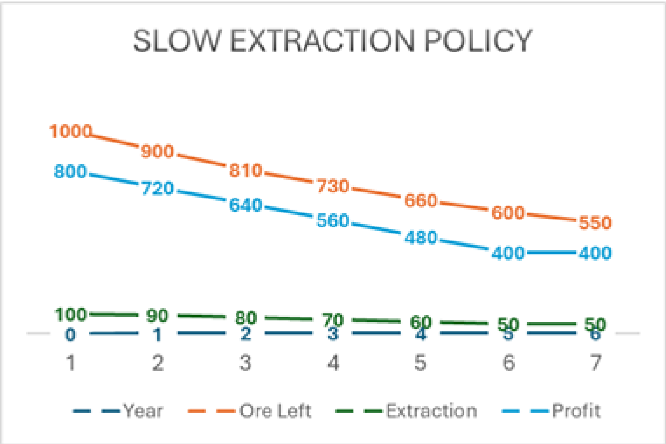

Figure 5: Although we have a long mine life the cummulative profit generated is K4000.00 which is low, hence under-utilizing resources.

Discussion

For the deterministic model and the stochastic model (Figure 1 and 2), in more practical mining operations, the rate of extraction which is denoted “k” is estimated with the help of historical production data, capacity of equiptment and operations efficiency (Rademeyer, Minnitt, & Falcon, 2019). It mirrors the rate that ore is being removed in relation to its remaining reserves. While on the other hand we have the randomness parameter denoted “σ” is derived from the variables that are observed in real world data including failures of equipment, geological uncertainty and factors to do with the environment like weather conditions. σ is estimated through statistical analysis of extraction records in the past (Jiang & Dimitrakopoulos, 2024).

The simulation of the optimization (Figure 3,4 and 5) determines the more profitable approach by comparing different extraction strategies. The results illustrate that aggressive extraction gives high operational cost and more rapid depletion of ore, while on the other hand slow extraction shows underutilized available resources. The balanced extraction rate turns out to be the best strategy for extracting ore, in which ore is being extracted at a controlled rate (MacNeil & Dimitrakopoulos, 2017) (Levinson & Dimitrakopoulos, 2020). This does not only maximize profit over time but maintains a reasonable mining lifespan. The importance of optimizing extraction rates is highlighted here rather that focusing on short term gains.

Conclusion

To model ore extraction processes in mining systems, a mathematical framework has been developed for these processes that are related to mining operations in Papua New Guinea. To describe and illustrate how ore depletes over time, under controlled environment, a predestined model on differential equations was firstly established while using Excel for simulations. An exponential trend shows how ore depletes where more ore is available extraction was fast and eventually slows down when resources run out.

To better reflect real world scenarios, the model was then extended by introducing a stochastic component (Øksendal, 2003). Uncertainties like equipment efficiency and geological variability are captured. This resulted in the model showing more realistic representation in mining.

Lastly to determine the most effective extraction strategy we introduce an optimization framework (Goodfellow & Dimitrakopoulos, 2016). This shows that a more controlled and balanced strategy returns highest overall profit and maintaining mine lifespan at the same time.

Practical implications for mining operations are found in the course of this study. By applying mathematical models mining companies can therefore enhance decision making which they can manage risks better given the uncertainty faced by mining industries and operations. As a decision support tool or guideline, the models can support planning in extraction schedules and optimizing profit and estimating the mine lifespan. Due to the unpredictable region that Papua New Guinea lies, stochastic modelling is important in particular for sustainability and reliability

By incorporating real mining data, the model will be improved to enhance accuracy and reliability. Later it can be extended and fine tuned to include multiple mine sites, different exchange rates and detailed functions in costs. For improvements in prediction and making of decisions, machine learning could be integrated as an advanced technique.

References

1. Githiria, J. (2019). Review of mathematical models applied in open-pit mining. International Symposium on Mine Planning & Equipment Selection, (pp. 92–102).

2. Goodfellow, R. C., & Dimitrakopoulos, R. (2016). Global optimization of open pit mining complexes with uncertainty. Applied Soft Computing, 40, 292–304.

3. Jiang, Y., & Dimitrakopoulos, R. (2024). Simultaneous stochastic optimisation of mining complexes with equipment uncertainty: Application at an open-pit copper mining complex. Mining Technology, 133, 241–256.

4. Levinson, Z., & Dimitrakopoulos, R. (2020). Simultaneous stochastic optimisation of an open-pit gold mining complex with waste management. International Journal of Mining, Reclamation and Environment, 34, 415–429.
5. MacNeil, J. A., & Dimitrakopoulos, R. G. (2017). A stochastic optimization formulation for the transition from open pit to underground mining. Optimization and Engineering, 18, 793–813.
6. Øksendal, B. (2003). Stochastic differential equations. In Stochastic differential equations: an introduction with applications (pp. 38–50). Springer.
7. Oksendal, B. (2013). Stochastic differential equations: an introduction with applications. Springer Science & Business Media.
8. Rademeyer, M. C., Minnitt, R. C., & Falcon, R. M. (2019). A mathematical optimisation approach to modelling the economics of a coal mine. Resources Policy, 62, 561–570.
9. Ross, S. M. (2014). Introduction to probability models. Academic press.
10. Zhang, J., Ruan, G., Bai, Y., & Ning, T. (2022). Mathematical Model and Numerical Simulation Study of the Mining Area with Multiple Air Leakage Paths. Mathematics, 10, 2484.