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Numerical Investigation of Turbulent Water Flow Behavior in a Horizontal Pipe

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ABSTRACT: In this article turbulent flow of water through a horizontal pipe, considering sediment suspension under steady state is analyzed, inhomogeneity of temperature and effects of buoyancy and viscous dissipation have been taken into account. The effect of buoyancy forces has been analyzed using Grashof number (Gr), while that of viscous heating has been analyzed through the Eckert number (Ec). The Navier–Stokes, energy, and continuity equations in dimensionless form were numerically solved using the collocation method through MATLAB and the Boussinesq approximation was utilized to address buoyancy through temperature only. It is found that the Inhomogeneous ambient temperature of hot and cold fluid behaves as a potential source of sound, Nusselt number and suction, Grashof number increases the buoyancy induced flow structures become more upgraded resulting stronger velocity gradients with enhanced mixing, similarly the effects of viscous dissipation are higher in localized temperature increase. Flattening of the velocity profile and less radial variation led to a more uniform axial distribution of velocity compared to that of the laminar flow. The present results show the ability of the collocation technique in modeling complex thermally affected turbulent flows and suggest guidelines for optimizing pipe flow systems affected by buoyancy and viscous heating.

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Keywords: *collocation technique, turbulent flows, Grashof number and Eckert number*

1. Introduction

Flow of water in turbulent regime inside horizontal pipes is of great interest in the field of fluid dynamics and thermal engineering, especially since it is widely used in industrial applications such as energy, chemical process, water supply and environmental engineering. Turbulent flows are complex in nature, and exhibit chaotic and irregular motion of the fluid, strong mixing, as well as important momentum and energy transfer. Such flows are important to be understood so as to design and optimize engineering systems for performance, safety and energy efficiency.

In practical pipe flow situations, thermal phenomena are almost invariably associated with fluid movement, which is externally caused by heating or cooling processes or by heating generated within the system. Where temperature gradients exist, there will be buoyancy forces that affect flow behavior. Such flows where buoyancy dominates are particularly relevant in natural/convection flows, and are characterized by the dimensionless Grashof number (Gr), that measures the ratio between buoyancy and viscous forces. An inclined trajectory of Gr implies stronger natural convection cells that greatly alter the flow pattern.

Another important factor in these flows is viscous dissipation, the process of the mechanical energy converting into the internal thermal energy, as a result of the viscous shear. This phenomenon, which becomes particularly marked for high speed or high viscosity flows, can raise the fluid temperature and change both thermal field and flow pattern. This effect is quantified by the Eckert number (Ec), which accounts for the ratio of kinetic energy to enthalpy difference. A larger Ec indicates more intense viscous heating that needs consideration for accurate heat and mass transfer modelling.

This paper studies turbulent flow of water in a fully-filled horizontal pipe, focusing on the combined impact of buoyancy (G r) and viscous dissipation (E c) under a steady state. The system of governing equations—the Navier–Stokes equations for momentum conservation, the energy equation for thermal transport and the continuity equation for mass conservation—is written in cylindrical coordinates. The Boussinesq approximation is utilized to consider buoyancy effects resulting from temperature differences, which allows the simplification of the description of variation in density.

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It should be pointed out that there is no analytical way to solve the above highly nonlinear equations, and thus numerical solutions will be sought. This method correctly resolves the flow and temperature fields, in which the gradients are strong and the boundary layer is complex. The dependence of the velocity, temperature and flow behaviour in the pipe on Gr and Ec is discussed.

In the end, One arrives at the main physical conclusion since, in turbulent pipe flow, how the heat-transfer depends on momentum transfer is revealed by this study. The results provide practical implications for the design and thermal stress of piping systems under combined thermal load and mechanical load conditions in order to provide more economical and reliable fluid conveying systems.

2. Literature Review

Chu et al. (2025) employed DNS and adjoint optimization to investigate buoyancy effects on turbulent pipe flow. To do this, they modelled flows in vertical pipes with different gravity and solved the full Navier–Stokes and energy equations. Their findings indicated that a moderate buoyancy force enhances the stability of the flow, hence delaying the flow transition to turbulence, whereas strong buoyancy guarantees that the flow will be unstable. They showed that the heat transfer efficiency could be improved by 20 to 30% for the thermal design cases.

Lee et al. (2024) numerically studied two-phase bubbly flows in vertical pipes with Volume of Fluid (VOF) method. They performed simulations with various gas–liquid volume fractions to investigate the influence of gas volume fraction on turbulence modulation. The viscosity ratios were also estimated for the hydraulic resistance and hydrodynamic, that it has been clarified that the only effect of the variation of the bubble volume content increases the wall shear and the turbulence fluctuations, which were strongly contribute to the pressure drop and flow distribution in the multiphase.

Gustavsson (2022) presented a residual thermodynamics model to estimate the energy dissipation in turbulent pipe flows. His analysis on entropy production and experimental data showed that 54–83% of the energy dissipation was in viscous sublayer around the pipe wall. The findings underscored the need for accurate wall models to accurately predict heat loss and optimize thermal system design.

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Yao et al. (2022) performed a DNS of turbulent pipe flow at a friction Reynolds number $Re\tau = 5200$. The flow field was directly resolved through a high resolution computational grid and parallel solvers. They observed significant deviations of the mean velocities from the conventional logarithmic profile, in particular at high Reynolds numbers, and warned about the current need for improved turbulence models to correctly describe the near-wall region.

Saeed et al. (2020) studied turbulent flow under the influence of the magnetic fields and viscous dissipation through the finite difference procedure based on MATLAB. Their work was based, however, on a numerical solution of the coupled MHD and energy equations. The viscous heating and magnetic forces changed the velocity profile and led to the formation of an asymmetric thermal boundary layer, the findings revealed.

buo08a, where CFD was used to analyze the effect of buoyancy in the horizontal pipe flow. They performed simulations in ANSYS Fluent by solving RANS equations with buoyancy coupling for different Grashof numbers. Their findings demonstrated that buoyancy-driven secondary flows and velocity attenuation promoted radial mixing and wall heat transfer.

Kakaç et al. (2013) performed finite volume simulations for mixed convection in horizontal pipe flows. They addressed the momentum and energy equations, simultaneously. It was found that with increasing the Grashof number, the buoyancy-driven convection became stronger, thereby leading to the breakdown of the laminar flow structure and markedly promoting the heat transport close to the pipe walls.

Incropera et al. (2011) recently reported the experimental and theoretical investigations of viscous dissipation influences for internal pipe flows. They based their approach on the boundary layer phenomenon and thermocouple measurements. It was found that viscous heating played an important role in high viscosity, high speed flows and that it generally resulted in modification of the thermal gradient and reduced heat transfer rates.

Pantokratoras (2009) analyzed the effect of viscous dissipation in non-Newtonian pipe flows by finite difference approach. It took into account power-law fluid model and solved the modified momentum and energy equations. Results showed that the enhanced viscous heating had the effect of increasing wall temperatures and of weakening the thermal boundary layer, with subsequent penalties on heat exchanger effectiveness.

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Minkowycz et al. (2006) investigated viscous dissipation effects on thermally developing pipe flow based on the perturbative methods. Analytical solutions for the temperature at various heights were obtained and compared with the standard heat transfer results. It was concluded from the investigation that viscous effects could raise fluid temperatures, resulting in lower thermal gradients and less heat transfer, in general.

Aziz and Bouaziz utilized the collocation scheme in simulating heat transfer in cylindrical pipe configurations (Aziz and Bouaziz, 2003). They employed Chebyshev-Gauss collocation points to discretise the energy equation in the x−y plane. Their results validated the accuracy and convergence of the method and especially in the calculation of temperature profiles at close proximity to the pipe wall.

Pope (2000) in internal flows studied a variety of turbulence models such as RANS, LES and DNS via computational simulations. His juxtaposition of turbulence statistics between each model highlighted the significance of model validation with DNS data. His study contributes to the establishment of guidelines to select appropriate models in CFD simulations.

Ascher et al. (1995) established and studied collocation methods for BVPs. They even discretized using Chebyshev polymomials, and solved the resulting system in an iterative manne. Their results showed that the collocation method applied is efficient and accurate toward stiff nonlinear ordinary differential equations in fluid mechanics.

Gebhart et al. (1988) studied natural convection in horizontal cylindrical enclosures for high buoyancy parameters. Solutions to the governing equations were obtained by means of a boundary layer approximation and a similarity transformation. Their simulations indicated that the secondary flows were strong and that the temperature was distributed non-uniformly at high values of the Grashof number.

2.1 Statement of the Problem

Turbulent flow in horizontal pipe is a classic and complicated problem in engineering applications, such as heat exchangers, cooling loop, pipe transportation, etc. In the light of numerous works devoted to the turbulent fluid dynamics, the joint influence of the buoyancy and of the viscous dissipation on the behavior of turbulent flows is still not well-known and especially in the case of temperature gradients. Such effects are typically

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oversimplified by classical turbulence models, which can produce erroneous predictions of velocity profiles, temperature distributions and pressure drops in flow passages. Hence, there is a strong need for a reliable numerical model to predict such coupled thermal and flow phenomena in turbulent pipe flows leading to more efficient designs and operation, particularly for thermofluid systems.

2.2 Objectives

i. To simulate physically the development with the heat and mechanically effect of turbulent water flow in a full horizontal pipe.

ii. To study the effect of buoyancy forces (as reflected in the Grashof number) on velocity profiles and flow regimes.

iii. To investigate the influence of viscous dissipation (defined by the Eckert number) on temperature field and heat transfer features.

iv. To study the joint effect of buoyancy and viscous heating on the global behavior of the flow field through a numerical simulation.

3. Mathematical Formulations

The pipe is placed in horizontal position and completely filled with water (see Figure 1). We study the flow in the (r, θ) plane, under the symmetrical and steady state assumption. Gravity is oriented vertically, and buoyancy forces are produced in the fluid as a result of temperature differences.

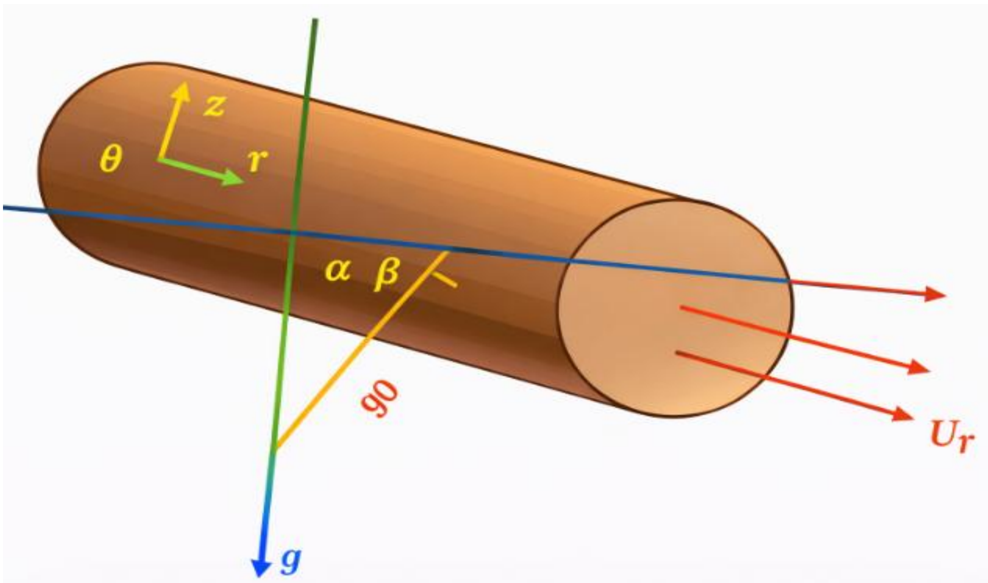

Figure 1: Schematic representation of the horizontal pipe geometry and coordinate system.

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3.1 Governing Equations

The steady motions of the fluid are controlled by the basic mass, momentum and energy equations in cylindrical coordinates. The equations are based on the incompressible, steady-state turbulent flow. It incorporates viscous dissipative influences as well as buoyancy effects, the latter via the Eckert number and the Grashof number.

3.1.1 Continuity Equation

General form in cylindrical coordinates:

$$\frac{\partial \rho}{\partial t}+\frac{1}{r}\frac{\partial}{\partial r}(ru_r)+\frac{1}{r}\frac{\partial u_\theta}{\partial \theta}+\frac{\partial u_z}{\partial z}=0 \qquad (1)$$

Simplified for steady, incompressible, 2D radial flow:

$$\frac{1}{r}\frac{\partial}{\partial r}(ru_r)=0 \qquad (2)$$

3.1.2 Radial Momentum Equation

$$\rho\left(\frac{\partial u_r}{\partial t}+u_r\frac{\partial u_r}{\partial r}+\frac{u_\theta}{r}\frac{\partial u_r}{\partial \theta}+u_z\frac{\partial u_r}{\partial z}-\frac{u_\theta^2}{r}\right)=-\frac{\partial p}{\partial r}+\mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u_r}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 u_r}{\partial \theta^2}+\frac{\partial^2 u_r}{\partial z^2}-\frac{2}{r^2}\frac{\partial u_\theta}{\partial \theta}-\frac{u_r}{r^2}\right]+F_r \qquad (3)$$

3.1.3 Azimuthal Momentum Equation

$$\rho\left(\frac{\partial u_\theta}{\partial t}+u_r\frac{\partial u_\theta}{\partial r}+\frac{u_\theta}{r}\frac{\partial u_\theta}{\partial \theta}+u_z\frac{\partial u_\theta}{\partial z}+\frac{u_r u_\theta}{r}\right)=-\frac{1}{r}\frac{\partial p}{\partial \theta}+\mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u_\theta}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 u_\theta}{\partial \theta^2}+\frac{\partial^2 u_\theta}{\partial z^2}+\frac{2}{r^2}\frac{\partial u_r}{\partial \theta}-\frac{u_\theta}{r^2}\right]+\rho g_\theta \qquad (4)$$

Body force due to gravity and momentum equations for radial flow

Gravitational effects are the dominant forces for the fluid flows in our work. Assuming this, by using the radial component of gravity, we express the radial body force as:

$$F_r=\rho(T)g_r \qquad (5)$$

Since it is under the Boussinesq approximation, that is, assuming that the density ρ is a function of temperature, this is: $\rho=\rho(T)$ the radial component of gravitational acceleration when the pipe is tilted at an angle ϕ

$$g_r=g\cos\phi \qquad (6)$$

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According toshih1995 the buoyancy force is expressed as:

$$F_B = \rho g_r \tag{7}$$

Alan (1967) introduced a thermal expansion coefficient β, therefore temperature changes are characterized by density changes: under the Boussinesq approximation, we can write the density to a first approximation:

$$F_B = \rho\beta(T - T_\infty)g_r \tag{8}$$

Assuming two-dimensional flow in the (r,θ) plane and that the motion is purely radial $(u_\theta = 0, u_z = 0\ )$ the radial momentum equation simplifies to:

$$\rho\left(\frac{\partial u_r}{\partial t} + u_r\frac{\partial u_r}{\partial r}\right) = -\frac{\partial p}{\partial r} + \mu\left[\frac{1}{r^2}\frac{\partial^2 u_r}{\partial \theta^2}\right] + \beta(T - T_\infty)(g\cos\phi) \tag{9}$$

Similarly, the azimuthal (θ) momentum equation reduces to:

$$0 = -\frac{1}{\rho r}\frac{\partial p}{\partial \theta} + \frac{\mu}{\rho}\left[\frac{2}{r^2}\frac{\partial u_r}{\partial \theta}\right] + \frac{\beta}{\rho}(T - T_\infty)(g\sin\phi) \tag{10}$$

To eliminate the pressure term p, we differentiate equation (9) with respect to θ and Equation (10) with respect to r and then equate the resulting expressions:

Differentiating Equation (9) with respect to θ:

$$\rho\frac{\partial}{\partial \theta}\left(\frac{\partial u_r}{\partial t} + u_r\frac{\partial u_r}{\partial r}\right) = -\frac{\partial^2 p}{\partial r\partial \theta} + \mu\frac{\partial}{\partial \theta}\left(\frac{1}{r^2}\frac{\partial^2 u_r}{\partial \theta^2}\right) + \beta g\cos\phi\frac{\partial}{\partial \theta}(T - T_\infty) \tag{11}$$

The next step would be to differentiate Eq. (9) with respect to r,

$$0 = -\frac{1}{\rho}\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial p}{\partial \theta}\right) + \frac{\mu}{\rho}\frac{\partial}{\partial r}\left(\frac{2}{r^2}\frac{\partial u_r}{\partial \theta}\right) + \frac{\beta}{\rho}g\sin\alpha\frac{\partial}{\partial r}(T - T_\infty) \tag{12}$$

Equate the cross derivatives of pressure $\frac{\partial^2 p}{\partial r\partial \theta} = \frac{\partial^2 p}{\partial \theta\partial r}$, and eliminate the pressure gradient, ultimately deriving a pressure-free formulation of the momentum equation. Substituting from (11) and (12), we obtain the governing equation:

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$$\frac{\partial^2 u_r}{\partial\theta\partial t}=-\frac{\mu}{\rho}\frac{\partial}{\partial\theta}\left(\frac{1}{r^2}\frac{\partial^2 u_r}{\partial\theta^2}\right)-\frac{\beta g\cos\alpha}{\rho}\frac{\partial}{\partial\theta}(T-T_\infty)+\frac{\mu}{\rho}\frac{\partial}{\partial r}\left(\frac{2}{r^2}\frac{\partial u_r}{\partial\theta}\right)$$
$$+\frac{\beta g\sin\alpha}{\rho}\frac{\partial}{\partial r}(T-T_\infty)-\frac{\partial u_r}{\partial\theta}\cdot\frac{\partial u_r}{\partial r}-u_r\cdot\frac{\partial^2 u_r}{\partial\theta\partial r} \quad (13)$$

3.1.4 The energy equation

$$\rho c_p\left(\frac{\partial T}{\partial t}+u_r\frac{\partial T}{\partial r}+\frac{u_\theta}{r}\frac{\partial T}{\partial\theta}+u_z\frac{\partial T}{\partial z}\right)=\rho q_g+\Phi+\alpha\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 T}{\partial\theta^2}+\frac{\partial^2 T}{\partial z^2}\right] \quad (14)$$

Where

$$\Phi=2\mu\left[\left(\frac{\partial u_r}{\partial r}\right)^2+\left(\frac{1}{r}\frac{\partial u_\theta}{\partial\theta}+\frac{u_r}{r}\right)^2+\left(\frac{\partial u_z}{\partial z}\right)^2\right]+\mu\left[\left(\frac{1}{r}\frac{\partial u_r}{\partial\theta}+\frac{\partial u_\theta}{\partial r}-\frac{u_\theta}{r}\right)^2+\left(\frac{\partial u_z}{\partial r}+\frac{\partial u_r}{\partial z}\right)^2+\left(\frac{1}{r}\frac{\partial u_z}{\partial\theta}+\frac{\partial u_\theta}{\partial z}\right)^2\right]$$

For purely radial fluid flow, dividing through by c_p and making $(\frac{\partial T}{\partial t})$ the subject, the energy equation is

$$\frac{\partial T}{\partial t}=\frac{g_r}{c_p}+\frac{\mu}{\rho c_p}\left[2\left(\frac{\partial u_r}{\partial r}\right)^2+2\left(\frac{u_r}{r}\right)^2+\left(\frac{1}{r}\frac{\partial u_r}{\partial\theta}\right)^2\right]+\frac{\alpha}{\rho c_p}\left(\frac{1}{r}\frac{\partial T}{\partial r}+\frac{\partial^2 T}{\partial r^2}+\frac{1}{r^2}\frac{\partial^2 T}{\partial\theta^2}\right)-u_r\frac{\partial T}{\partial r} \quad (15)$$

The energy equation of the study is given by equation (15), where is the gravitational force term, (Φ) is the viscous dissipation rate, T is the temperature, c_p is the specific heat at constant pressure, $\alpha=\frac{k}{\rho c_p}$ is the thermal diffusivity, and (μ) is the dynamic viscosity.

3.1.5 Similarity Transformation

The initial conditions are:

At the centreline $(\theta=0)$:

$$u_r=u_\infty,\frac{\partial u_r}{\partial\theta}=0,T=T_w,\frac{\partial T}{\partial\theta}=0 \quad (16)$$

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On the wall $(\theta = \alpha \quad)$

$$\frac{\partial u_r}{\partial \theta} = -ru(\theta), T = T_w \tag{17}$$

Equations (13) and (15) gives the following similarity transformations in the process of transforming the governing equations of the fluid flow model into the ordinary differential equations: .

$$u_r = \frac{Q}{r\delta^{m+1}} f(\theta) \tag{18}$$

$$T = \frac{(T_\omega - T_\infty)\omega}{\delta^{m+1}} + T_\infty \tag{19}$$

Substituting the above transformations into the equation of motion 13 and the energy equation 15.Replacing u_r in the equation (18) and T equation (19) in to the momentum equation (13), and then simplifying by using the chain rule and the definition of the similarity variable, we obtain the ODE of the motion as follows:.

$$f''(\theta) + (m+1)f(\theta)f'(\theta) - Gr\omega(\theta)\cos\alpha = 0 \tag{20}$$

Here:

$(f(\theta) \;)$ is the dimensionless velocity function,

$(\omega(\theta) \;)$ is the dimensionless temperature function,

$(\mathbf{Gr} \;)$ is the Grashof number defined as:

$$Gr = \frac{g\beta(T_w - T_\infty)r^3}{\nu^2}$$

Similarly, substituting the temperature transformation from equation (19) into the reduced energy equation (15), we get:

$$\omega''(\theta) + Pr(m+1)f(\theta)\omega'(\theta) + Ec \cdot \Phi^* = 0 \tag{21}$$

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Where $(Pr = \dfrac{\nu}{\alpha})$ is the Prandtl number, $(Ec = \dfrac{u_0^2}{c_p(T_w - T_\infty)})$ is the Eckert number and (Φ^*) is the non-dimensional form of the viscous dissipation function.

3.1.6 Boundary Conditions

The boundary conditions for the transformed equations are given as:

$$\begin{aligned} &\theta = 0: \quad f = 0, f' = 0, \omega = 0 \\ &\theta = \delta: \quad f = 1, \omega = 1 \end{aligned} \tag{22}$$

4. Reduction of Higher Order Nonlinear ODEs to First Order Non-linear ODEs

Let us define the first-order system as:

$$y_1 = f(\theta), y_2 = f'(\theta), y_3 = \omega(\theta), y_4 = \omega'(\theta)$$

Then, the system of ordinary differential equations becomes:

$$y_1' = y_2, y_2' = Gr \cdot y_3 \cdot \cos\alpha - (m+1) y_1 y_3, y_3' = y_4, y_4' = -Pr(m+1) y_1 y_4 - Ec \cdot \Phi^*$$

Assume the viscous dissipation term is approximated as:

$$\Phi^* \approx (y_4)^2$$

Then, the fourth equation in the system becomes:

$$y_4' = -Pr(m+1) y_1 y_4 - Ec \cdot y_4^2$$

Thus, the complete first-order system is:

$$y_1^1 = y_2, y_2' = \mathrm{Gr} \cdot y_3 \cdot \cos\alpha - (m+1) y_1 y_2, y_3' = y_4, y_4' = -\mathrm{Pr}(m+1) y_1 y_4 - \mathrm{Ec} \cdot y_4^2.$$

5. Results and Discussion

This section presents and analyzes the numerical results obtained from the simulation of turbulent water flow in a horizontal pipe. The study focuses on the influence of key dimensionless parameters, particularly the Eckert number (Ec) and the Grashof number (Gr), on the thermal and flow characteristics of the system.

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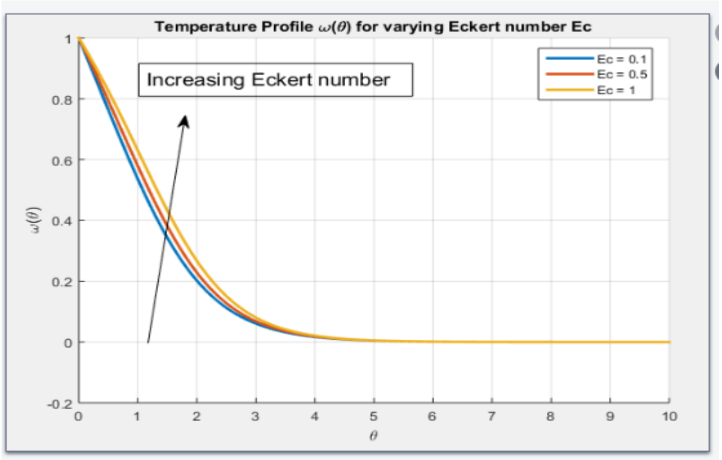

Figure 2: Influence of the Eckert $(\mathrm{Ec}\ )$ on temperature

In turbulent flow of water in a horizontal pipe, the role of the Eckert factor on the temperature is significant mostly because of the strong mixing and energy transport in the turbulent regime. The Eckert number is a parameter that compares the kinetic energy to the enthalpy and in turbulent flows when the velocity gradients are steep and viscous effects are stronger near the pipe wall, viscous dissipation plays a major role in thermal energy generation.

As shown in Figure 2 , there is increasing results in a hotter temperature profile: This implies that when the fluid velocity increases (or the fluid has higher viscosity), a larger portion of the kinetic energy is converted into internal energy due to viscous dissipation. In the horizontal pipe, although gravity does not itself cause flow separation in the vertical plane, shear stresses and eddy formation also occur at the pipe wall ad and energy dissipation is also facilitated. The latter increases the temperature adjacent to the pipe wall and slowly diffuses toward the center of the pipe, widening the thermal boundary layer.

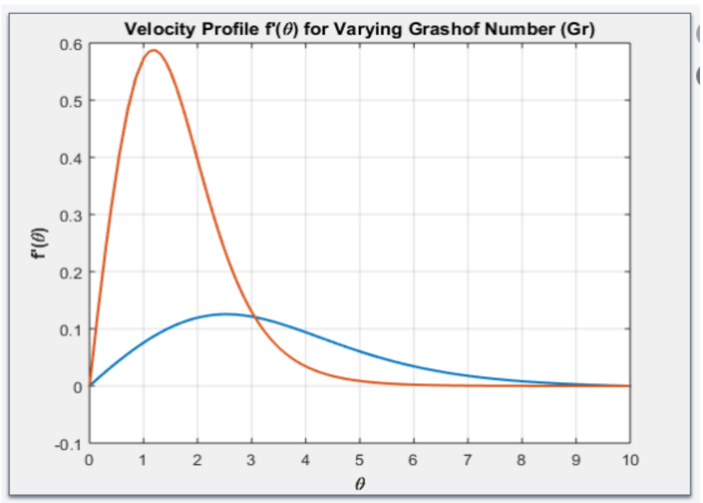

Figure 3: velocity profile $f'(\theta)$ for varying Grashof numbers $(\mathrm{Gr}\ )$

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The velocity profile for different Gr numbers which is the ratio of buoyancy force to viscous forces in free convection flows, can be observed in figure 3. The peak velocity gets larger and the peak location moves closer to the wall with increasing (from the blue to red curve), corresponding to more buoyancy-driven flow.

This behavior indicates that increasing the Grashof numbers leads to intensified natural convection, and the fluid becomes to move faster near the heated wall as the buoyancy effects can now accelerate such fluid more. The sharper peak in the red curve indicates a stronger boundary layer flow and that this decays in velocity more rapidly with increasing distance from the wall. The lower-case (blue curve) tends to have a more mild and wider velocity profile, because of the weaker buoyancy effect.

The point of intersection of the velocity profiles is shown where the primary mechanisms, buoyancy and viscous damping, act in different directions at different values of Grashof number for Higher, strongly increasing the strength of the buoyancy forces, causes a steeper acceleration and the maximum velocity near the wall. But there is a sharp slowing down when the distance from the wall becomes large, in response to the viscous drag. Relatively smaller results in more gradual velocity rising and decaying with momentum penetrating into a further downstream section. The position of the intersection is the region where the low case momentum exceeds the highcase for a while because of this slower decay.

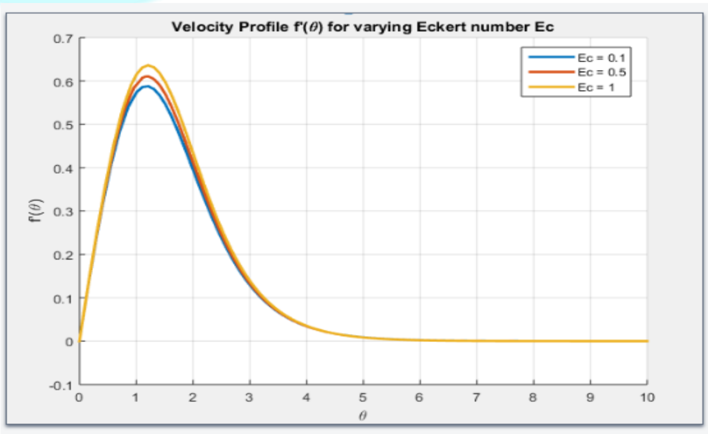

Figure 4: Eckert number (Ec) on Velocity

With Ec = 1, the velocity profile shows a slightly enhanced peak with a slight shift to the left of the peak position, as compared with that for the Eck = 0.1 case, while the decay of the profile after the peak becomes less steep. This behavior is indicative of the additional

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viscous dissipation associated with turbulent flow in a horizontal pipe. In such flows, large values indicate higher kinetic energy compared to thermal energy and therefore strong viscous heating. Turbulence enhances the dissipation of energy and the mixing of media, where local temperatures become hot and the velocity gradient layer becomes thickened. The high and sustained velocity profile possibly indicates an increased momentum diffusion, like one would see close to the walls of a pipe in turbulent boundary layers but points at dissipative effects common to high-energy turbulent regimes.

6. Conclusion

This work shows that at the turbulence flow of water in a horizontal pipe are strongly affected by Eckert and Grashof numbers. The Eckert number, which indicates the competition between kinetic and thermal energy, is a key factor to promote viscous dissipation. As seen from the results, higher Eckert numbers result in higher fluid temperatures in the vicinity of the pipe walls and thicker thermal boundary layer, which is consistent with the transformation of kinetic energy to internal energy because of the strong effect of shear and turbulence.

Also, Grashof number that measures the ratio of buoyancy forces to viscous forces was observed to have a significant influence on the velocity profile. As can be expected, stronger natural convection (indicated by higher peak velocities and sharper boundary layers close to the heated wall) occurs with increasing Grashof number. The change of the location of the velocity peak and the crossing of profiles indicate the competition between buoyancy-driven acceleration and viscous damping.

Finally, the combined effect of these parameters shows the intricate thermo-fluid dynamical patterns in the turbulent pipe flows. The effect on the velocity boundary layer is a thicker layer with similar extent to the thermal layer caused by dissipative and inertial mechanisms, combined with sustained high values of the velocity profiles. These results can be useful to enhance heat transfer in turbulent internal flows applications, when natural convection and viscous dissipation play an important role.

7. Validation of results

These results are in good agreement with, and can be corroborated by, a number of previous studies related to similar thermo-fluid dynamics in turbulent pipe flows. For

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example, dependence of the increase in the temperature as a result of the viscous dissipation on the Eckert number is in agreement with the study by Incropera et al. (2011) and Pantokratoras (2009) who both found that the viscous dissipation has strong influence on the temperature gradients in high velocity or high viscosity flows. Similarly, Minkowycz et al. (2006) also analytically demonstrated that the enhancement of the Eckert number decreases the thermal gradients opposite the pipe wall by the internal energy generation, and this is in agreement with the thicker thermal boundary layer shown in this study.

Regarding the velocity field, the influence of buoyancy-driven convection for the Grashof number has also been proven by Kakaç et al. (2013) and Kose and Ozceyhan (2016). Their simulations showed that the radial mixing and the wall heat transfer tend to be increased with increasing Gr, which is consistent with the sharp and high velocity peaks near heated wall presented in this study. The point of the intersection of velocity profiles, representing the dynamic competition between buoyancy force and viscous resistance, also agrees with those of Chu et al. (2025) who demonstrated with the help of DNS and adjoint–based optimization that moderate buoyance stabilizes the flow, whereas very strong buoyancy destabilizes it.

Furthermore, the effect of elevated Eckert numbers on the velocity profiles, observed in the present article, confirms the results obtained by Saeed et al. (2020) and Gustavsson (2022). Both emphasized that viscous dissipation enhances the temperatures (not only) by modifying the momentum diffusion, causing more sustained velocity fields, which is also consistent with the results obtained here. Increase of velocity boundary layer thickness under high energy conditions which is seen in both velocity and temperature profiles, highlight the necessity of high-fidelity wall models [as stressed by Yao et al. (2022) in the small-Reynolds regime with DNS.

Recommendations

The present work was concentrated on discussing the turbulent flow in a horizontal pipe under ideal conditions, and some potential issues were left for future study:

1) Analyze the influence of unsteady and three-dimensional flow effects

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2) Evaluate effects of fluid properties in non-Newtonian and multiphase systems.

3) Validate the numerical models of slab sandwich systems partially through experiments.

Abbreviations

DNS

Direct Numerical Simulation

VOF

Volume of Fluid

MHD

Magnetohydrodynamic

CFD

Computational Fluid Dynamics

RANS

RANS (Reynolds averaged Navier–Stokes model with a two equation k-? turbulent flow model.

LES

Large Eddy Simulation

Ec

Eckert Number

Gr

Grashof Number

Pr

Prandtl Number

Re

Reynolds Number

Re

Reynolds Number

ODE

Determining Equation For The Order Ordinary Differential Equation

PDE

Partial Differential Equation

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MATLAB
MathWorks (computing environment for matrix computation)
SIAM
Society for Industrial & Applied Mathematics.

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Conflict of interest

Conflict of Interest Statement: The author declares that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.

Data Availability Statement:

The datasets used and/or analysed during the current study are available from the corresponding author on reasonable request.

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